# Business Cycle during Structural Change: Arthur Lewis' Theory from a Neoclassical Perspective.*

Kjetil Storesletten Bo Zhao Fabrizio Zilibotti

April 1, 2019

**Abstract**

We provide a unified theory of business cycles and structural change. We document that the nature business cycle evolves over the process of development and structural change. In countries with large declining agricultural sectors, aggregate employment is uncorrelated with GDP. During booms, employment in agriculture declines while labor productivity increases in agriculture more than in other sectors. The focal point of the theory is the link between structural change, productivity shocks, and the speed of modernization of agriculture. The agricultural sector comprises a modern and a traditional (labor-intensive) subsector. As capital accumulates, agriculture becomes increasingly capital intensive as modern agriculture crowds out traditional agriculture. Structural change accelerates in booms and slows down in recessions. We estimate the model and show that it accounts well for both the structural transformation and the business cycle fluctuations of China.

*Storesletten: Department of Economics, University of Oslo, P.O Box 0317, Oslo, Norway, kjetil.storesletten@econ.uio.no. Zhao: National School of Development, Peking University, No.8 Yiheyuan Road, Haidian District, Beijing, China, zhaobo@nsd.pku.edu.cn. Zilibotti: Department of Economics, Yale University, fabrizio.zilibotti@yale.edu, 28 Hillhouse Ave., New Haven, United States. We thank seminar participants at mid-west macro conference 2017, European Economics Association Annual Meeting 2017, Brown University, University of Oslo, Tsinghua University, Fudan University, Shanghai University of Finance and Economics, Shanghai Jiao Tong University, East China Normal University, Central University of Finance and Economics.

# 1 Introduction

Economic fluctuations differ systematically across countries at different stages of development (see Acemoglu and Zilibotti (1997), Aguiar and Gopinath (2007)). For example, aggregate employment is uncorrelated with GDP in poor and middle-income countries, while highly procyclical in rich countries. These differences are manifested as countries undergo structural change from agriculture to nonagriculture. China provides a good example. The country has experienced an economic transformation where the share of employment in agriculture has fallen from more than 60% in 1980 to less than 30% today. This structural change has accelerated during economic booms. It follows that employment in nonagricultural sector has been procyclical while employment in agriculture has been strongly countercyclical. Consequently, aggregate employment has been uncorrelated with GDP and relatively smooth.

Structural change has been related to business cycles also in terms of labor productivity. As China's agriculture has shed workers to manufacturing, agriculture has experienced modernization and increased productivity. Interestingly, this process has occurred even at business-cycle frequencies: relative labor productivity in agriculture has increased precisely when workers have been leaving agriculture.

These features are not specific to China. We document that these and other characteristics of the business cycle are shared by the majority of countries at comparable stages of development. Extending the work of Da Rocha and Restuccia (2006) beyond OECD countries, we show that the correlation between agricultural employment and aggregate GDP varies systematically with the relative size of the agricultural sector. While employment in agriculture is procyclical in industrialized countries, it is countercyclical in economies with a large agricultural sector. At the same time, employment in nonagricultural sectors is strongly procyclical in all countries. Finally, downswings in agricultural employment are associated with upswings in the relative productivity and capital intensity of the agricultural sector in developing countries. Such correlations are absent in fully industrialized economies.

To rationalize these observations, we propose a theory where the economy is subject to stochastic productivity shocks. In our theory, the economic mechanisms driving the structural transformation are the same forces that determine different business cycle properties at different stages of development. More formally, we construct a three-sector model where growth and structural change are driven by heterogenous TFP growth at the sectoral level (as in Ngai and Pissarides (2007)) and endogenous capital accumulation (as in Acemoglu and Guerrieri (2008)). The crux of the theory is that investments and capital deepening generate an endogenous reallocation from the agricultural to the nonagricultural sector. The rural sector comprises two subsectors, modern and traditional agriculture, which produce imperfect substitutes using different technologies. In particular, modern agriculture uses capital and labor whereas the traditional sector uses no capital. Under empirically plausible assumptions about the elasticities of substitution, capital accumulation implies that, on the one hand, the share of agriculture in total GDP shrinks over time, while on the other hand agriculture undergoes a modernization process. Throughout transition, agriculture becomes more productive – both in an absolute sense and relative to the nonagricultural sector. The increase in relative labor productivity is due to a progressive downsizing of the traditional sector that works as a labor force reserve during the industrialization process. Modernization brings about an increase in the average capital intensity of agriculture. In the long run, the equilibrium converges to an asymptotic balanced growth path where the agricultural sector is small, modernized, and highly productive. These are typical features of industrialized economies.

The mechanism in our theory is reminiscent of Lewis (1954). As in that model, a labor intensive sector makes the supply of labor very elastic. In the original Lewis' theory, labor supply is infinitely elastic as long as the traditional sector exists. When the traditional sector becomes empty, the elasticity falls discretely being thereafter fully determined by the technology of the modern sector. In particular, the elasticity turns constant if the technology of the modern agriculture is a CES. In our model, instead, the elasticity of substitution between capital and labor declines gradually during the process of modernization of agriculture. A poor economy behaves like a Lewis economy; then, throughout the development process, it becomes more and more similar to a standard neoclassical economy. The changing elasticity between capital and labor has implication for business cycles. When the elasticity is high (i.e., when the agricultural sector is dominated by the traditional sector), the economy responds to shocks affecting the relative productivity across sectors by shifting many workers towards the more productive sectors (i.e., modern agriculture and nonagriculture) with limited effects on wages and on relative prices. When the traditional sector is small, this margin of adjustment is muted, and productivity shocks translate into smaller reallocation of labor across sectors. Wages and prices move more leading to larger swings in aggregate labor supply in response to shocks – similar to the mechanism in standard real business cycle models. We show that this model generates predictions in line with stylized facts about both structural change and business cycles that we document in Section 2.2.

We estimate the growth model using data from China. The key parameters are the elasticities of substitution between the output of the agricultural and nonagricultural sector and between that of the two agricultural subsectors. For comparability with previous estimates in the literature, we allow preferences to be nonhomothetic using a generalized Stone-Geary specification as in Herrendorf et al. (2013). A key finding is that the elasticity of substitution between the agricultural and nonagricultural good is larger than unity. This finding is consistent with the observation that the ratio between the expenditure share in agriculture and nonagriculture is positively correlated with the real GDP ratio between agriculture and nonagriculture. The deviations from nonhomothetic preferences are estimated to be very small. Our findings are in line with the recent evidence in Alvarez-Cuadrado and Poschke (2011).[1]

Having estimated the deterministic model, we introduce stochastic TFP shocks in order to study the business cycle properties of the model. The productivity shocks cause economic fluctuations and affect the speed of structural change. When the agricultural sector is large, positive productivity shocks in the nonagricultural sector speed up capital accumulation and labor reallocation out of agriculture. Interestingly, a positive productivity shock in the nonagricultural sector induces an even larger increase of labor productivity in agriculture. The reason is that most of the short-run reallocation of labor away of the agriculture is drawn from the traditional agricultural sector. Therefore, agriculture experiences a sharp increase in capital intensity and average labor productivity.

Finally, we compare the results with moments of the data using annual statistics from China. This step is technically challenging because we cannot rely on a standard approach of approximating the model in the neighborhood of a steady state. The reason is that as structural change is still ongoing, the economy is very far from a stochastic steady state. The moments from the

[1] Note that our estimates is different from that obtained by Herrendorf et al. (2013). They estimate a production function with three sectors, agriculture, manufacturing, and services, and find a low elasticity of substitution close to Leontief when using value-added data. Their estimate hinges on an assumption of symmetry, namely that the same elasticity of substitution is imposed across the three sectors. We show that if one relaxes the symmetry assumption the estimated elasticity of substitution between manufacturing and services is indeed close to zero, whereas the elasticity of substitution between agricultural and nonagricultural goods is larger than unity.

model must then be estimated by simulating a large number of trajectories approaching the steady state, and calculating moments from that.

The model is quantitatively consistent both with China's structural transformation and with the business cycle properties of China. Moreover, as productivity grows and structural change progresses, the business cycle properties of the model become increasingly similar to those of advanced economies. In particular, as the traditional sector shrinks, so does the scope for intersectoral labor mobility in response to productivity shocks. Therefore, in an industrialized economy labor reallocation at business cycle frequencies becomes insignificant, while the bulk of the adjustment occurs along the labor-leisure margin. Thus, aggregate employment becomes more procyclical and more volatile. In addition, when the economy has been fully modernized, the average productivity of labor in agriculture responds less to productivity shocks. As a result, our model predicts (under the estimated parameters) that the ratio between the average labor productivity in agriculture and manufacturing is procyclical for an economy in transition like China. This is consistent with the empirical evidence for China in recent decades. However, at a later stage of the development process this ratio turns countercyclical.

Our research contributes to the existing literature on structural change pioneered by Baumol (1967) which includes, among others, Kongsamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Buera and Kaboski (2009), Alvarez-Cuadrado and Poschke (2011), Herrendorf et al. (2013), Boppart (2014), Comin et al. (2015), Alder et al. (2018). In our model, exogenous TFP growth and capital accumulation induce transition away from agriculture. The properties of the transition are consistent with Acemoglu and Guerrieri (2008), although in their model there is no traditional sector. The closest theoretical contribution in the literature is the recent paper by Alvarez-Cuadrado and Poschke (2011). They study the properties of two-sector models where both sectors use capital and labor as inputs. The elasticity of substitution between capital and labor is assumed to be constant within each sector but can differ across sectors. While in their model there is no explicit distinction between a modern and a traditional agricultural sector, our model nests theirs if we assume that the modern sector uses no labor and that the capital share in the modern sector is 100%.[2] In the general case, our model allows the elasticity of substitution between capital and labor in agriculture to

[2] In our model, we assume a Cobb Douglas production function in the nonagricultural sector, while Alvarez-Cuadrado and Poschke (2011) allows for a more general CES technology. Cobb Douglas is for simplicity and can be easily generalized.

change over the process of development due to the reallocation between traditional and modern agriculture. An advantage of our specification is that, even when the elasticity of substitution between the agricultural and nonagricultural good is larger than unity (which is consistent with our estimate), the labor share in agriculture remains positive, while in the CES technology proposed by Alvarez-Cuadrado and Poschke (2011) it would fall to zero. In the US, the labor share in agriculture is about 30% with no downward trend. Finally, the two models have different quantitative properties at business cycle frequencies (Alvarez-Cuadrado and Poschke (2011) do not consider business cycle fluctuations).

Our work also complements the existing literature on business cycles pioneered by Kydland and Prescott (1982) and Long and Plosser (1983) by adding explicitly the endogenous structural change. The classical multi-sector model focus on the stable economic structures or abstract from growth, for example, Benhabib et al. (1991), Hornstein and Praschnik (1997), Hornstein (2000), Horvath (2000a), Christiano et al. (2001), Kim and Kim (2006), etc. We extends the standard business cycle model to account for the business cycles properties at different stage of development. Related researches on cross-country business cycles differences including Da Rocha and Restuccia (2006) and Aguiar and Gopinath (2007).[3]

The remainder of the paper is organized as follows. In section 2 we present the facts on business cycles and structural change for a group of countries over the world, including China and the US. In Section 3, we present a dynamic stochastic general equilibrium model that describes the process of growth and structural change in an economy with a declining agricultural sector. In Section 4, we estimate the key structural parameters of the deterministic version of the model using data from China. We show that the model matches accurately the process of structural change in China. In Section 5, we show that the model is consistent with salient business cycle features of economies undergoing a process of structural change that are different from standard business cycles in fully industrialized economies. The conclusions are summarized in Section 6, while the Appendix includes proofs.

[3]In particular, Da Rocha and Restuccia (2006) documents that among OECD countries, economies with larger agricultural sectors have smoother aggregate employment fluctuations and that agricultural employment is less correlated with aggregate GDP. Relative to their paper we show that this pattern holds up when extending the sample of countries to include a large number of countries, including very poor countries that are predominantly agrarian. Moreover, we document several additional salient differences in business cycle properties between poor and rich countries and show that business cycle properties of China are representative of countries with a similar share of agriculture.

# 2 Facts on Business Cycles and Structural Change

This section documents how agriculture and its interaction with nonagriculture have changed over time. The process of economic development is associated with a significant downsizing of the agricultural sector. In the US, a third of the workforce was employed at farms in 1900. This employment share fell below 2% by 2000. Today, the average employment in agriculture is 4.6% in OECD countries, which compares with 31.6% in non-OECD countries (World Development Indicators 2017). The cross-country correlation between the employment share of agriculture and log GDP per capita is $-0.84$.

## 2.1 Modernization of Agriculture

The relative decline of agriculture is accompanied by a number of economic transformations. We document here three facts that will be important for our theory. First, as employment in agriculture declines, the capital intensity in agriculture is rising faster than in the rest of the economy. Second, during this transformation the labor productivity grows faster in agriculture than in the rest of the economy. Third, as the share of real value added in agriculture falls, so does the relative price of agricultural goods.

**Capital deepening.** As the agricultural sector shrinks in size and employs a decreasing share of the workforce, it undergoes a process of modernization involving capital deepening and labor productivity growth. While capital deepening is pervasive across all sectors, it is especially pronounced in agriculture: both the capital-output ratio and the capital-labor ratio grow faster in agriculture than in the rest of the economy. Second, while labor productivity grows in all sectors, it grows faster in agriculture than in other sectors. In economies with a large employment share in agriculture, the productivity gap between agriculture and nonagriculture is typically very large. This gap shrinks as economies develop.

Consider capital deepening. Figures 1-2 show the capital-output (K-Y) ratio in agriculture relative to the aggregate K-Y ratio over the process of economic development. Figure 1 (panel a) shows that the relative K-Y ratio (K-Y ratio in farm sector over the aggregate K-Y ratio) increased from about 40% in the pre-war period, to about 120% since the 1980s, indicating a sharp process of modernization of agriculture over time. The cross-country evidence shows a consistent picture: the relative (agriculture-to-aggregate) K-Y ratio is significantly lower in developing than in industrialized countries both in absolute and relative terms. Figure 2 (panel b) plots the relative K/Y ratio against the employment share of agriculture for the period 1995-2016. For example, that ratio is very low in Sub-Saharan African countries, where the agricultural sector is still very labor intensive and still employs a large proportion of the labor force. The regression coefficient is negative and highly significant. Since data are available over a 22 year panel, we can also study the within-country coevolution of employment shares in agriculture and relative K-Y ratios. To this aim, we split the sample for each country into two observations, the average for the period 1995-2005 and the average for the period 2006-2016, and regress the relative K-Y ratio on the employment share of agriculture and a full set of country dummies. This is identical to running a regression on growth rates. The results (reported in the appendix) show a significant negative relationship between employment in agriculture and relative capital intensive, consistent with the cross-country pattern.

**Labor productivity gap.** Capital deepening and technical change bring about higher labor productivity in all sectors of the economy. However, this growth is larger in agriculture than in the rest of the economy. Consistent with this observation, we observe a decreasing labor productivity gap between nonagriculture and agriculture. Figure 1 (panel b) shows that the labor productivity gap between nonfarm and farm sector declines over time in the US[4]. Figure 2 (panel b) shows a similar pictures across countries: the labor productivity gap in agriculture is especially high in developing countries.

The trend in the relative sectoral labor income share mirrors that of the labor productivity gap.[5] Figure 1 (panel c) shows that in the US the labor-income share in the farm sector relative to that of the nonfarm sector fell over time after World War II. Unfortunately, it is difficult to find comparable data across a large number of countries since the labor income share in agriculture is often poorly measured.

**Relative price of agricultural goods.** Finally, we are interested in the comovement of relative prices and quantities in agriculture. This comovement is informative about the elasticity of substitution between the products of agriculture and nonagriculture that will be a key parameter in our theoretical model. We use data for value added at current and constant prices.

[4]Labor productivity is defined as the value added per worker in current prices.

[5]We discuss below the link between the relative labor income shares and the labor productivity gap.

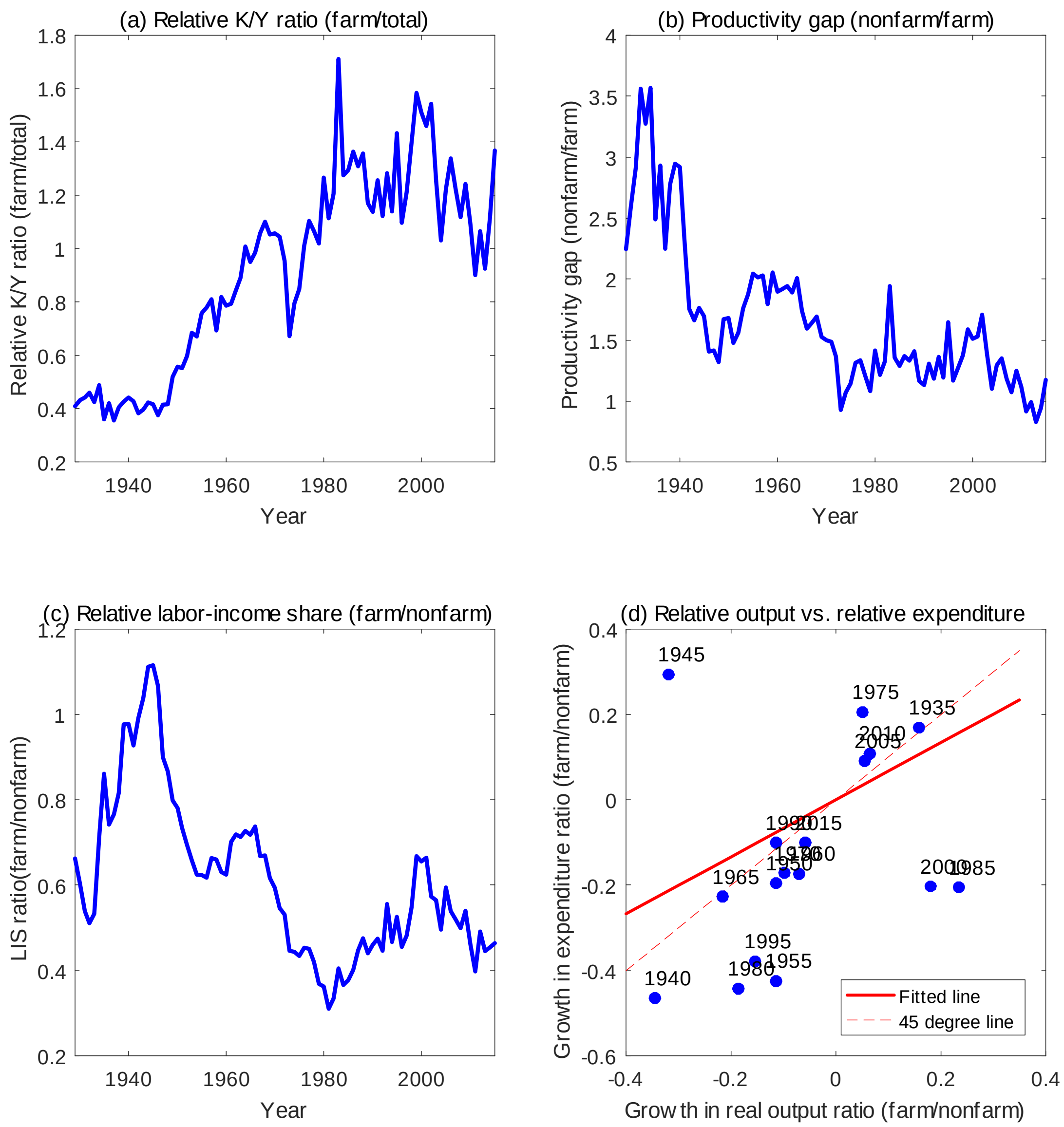



Figure 1: Panel a plots the farm capital-output ratio divided by the total capital-output ratio in the US. Panel b plots the Labor productivity gap over time in the US. The labor productivity gap is measured by the nonfarm value added per worker divided by the farm value added per worker. Panel c plots the labor income share (LIS) ratio in the US. The graph plots labor's income share in the farm sector divided by labor's income share in the nonfarm sector. We compute the labor's income share as the compensation of employees divided by the value-added output excluding the proprietors' income. Panel d plots the growth in relative (current price) value added vs. growth in relative output. The horizontal axis plots the 5-year growth in the relative value added (farm vs. nonfarm) at current prices. The vertical axis plots the annual growth in the relative real output (farm vs. nonfarm). Source: Capital stocks by sectors 1929-2015 are from the U.S. Bureau of Economic Analysis (BEA) Fixed Asset Tables 6.1 "Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods". The value-added output by sectors come from the National Income and Product Accounts (NIPA) Table 1.3.5 "Gross Value Added by Sector". Employment by sectors is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. Proprietors' income by sectors come from the NIPA Table 1.12. Compensation of employees by sectors come NIPA Table 6.2A, 6.2B, 6.2C, and 6.2D. The real sectoral value added is from NIPA Table 1.3.6. "Real Gross Value Added by Sector, Chained Dollars."

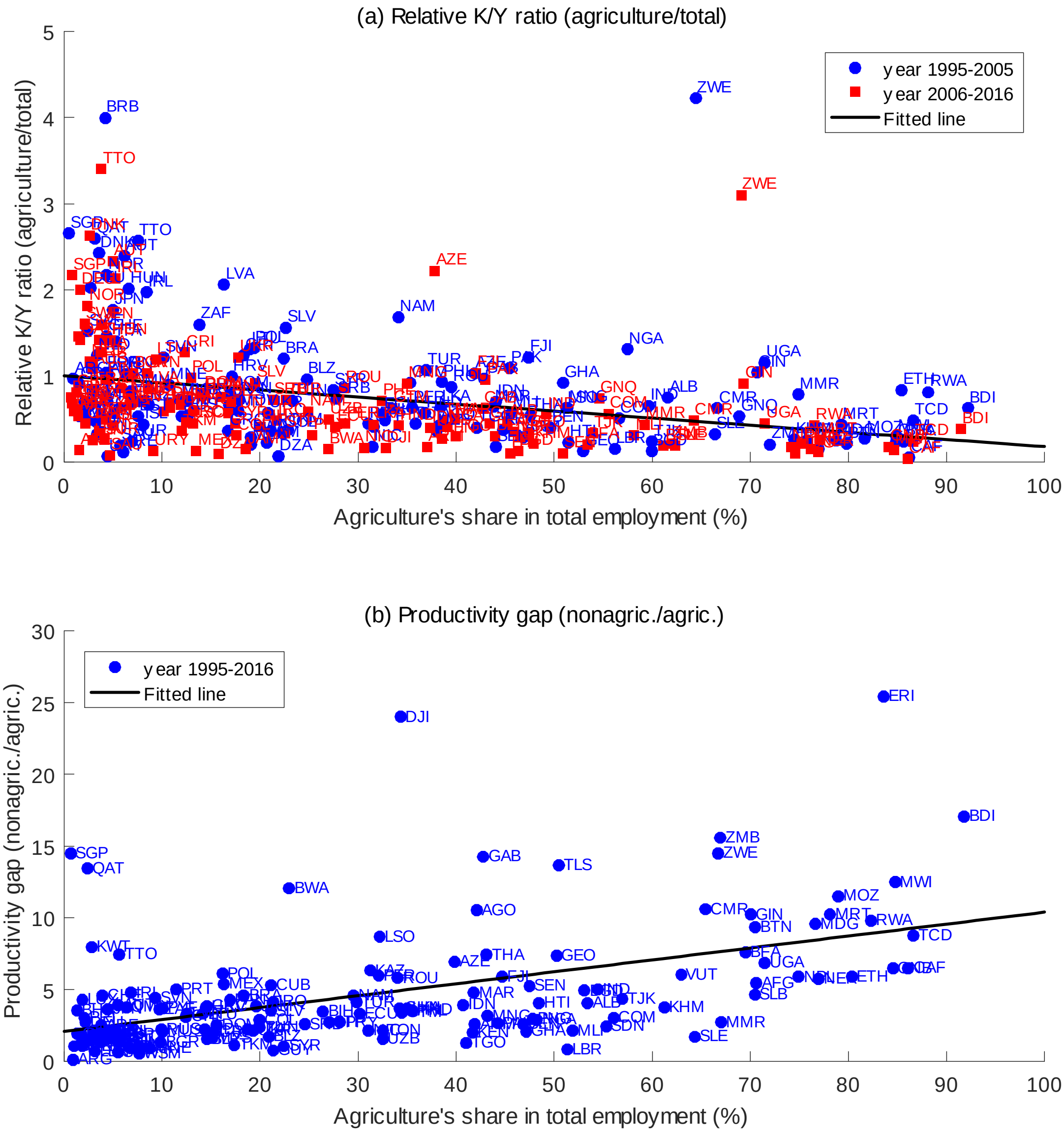



Figure 2: Panel (a) plots the relative capital-output ratio (agriculture vs. total) $\left(K^G/Y^G\right)/(K/Y)$ vs. the average agricultural employment share across countries. Each country has two observations: the average for the period 1995-2005 and the average for the period 2006-2016. Panel (b) plots labor productivity gap across countries. The labor productivity gap is measured by the nonagriculture value added per worker divided by the agriculture value added per worker. The horizontal axis shows the average employment share of agriculture over the sample period for each country. Source: FAO ($K^G$ is measured by the net capital stock; $Y^G$ is value added, both at current prices) and Penn World Table (capital stock and real GDP at current PPP). Agriculture's employment share comes from ILO modeled estimates. Agriculture's value added output share comes from World Bank database.

More formally, let $VA_G \equiv P_G \times Y_G$ and $VA_M \equiv P_M \times Y_M$ denote the value added in agriculture and nonagriculture, respectively, where $P_G$ and $P_M$ are sectoral GDP deflators. Figure 1 (panel d) plots the 5-year growth in the relative value added (at current prices) $VA_M/VA_G$ against the 5-year growth in the relative real output $Y_M/Y_G$ for the US. The figure shows a highly significant positive correlation. The finding is robust to using 1-year and 10-year growth rates instead of 5-year growth rates. Figure 13 in the appendix show that this correlation is strongly positive also in China.

## 2.2 Business Cycle

In this section, we document that the process of development goes hand-in-hand with a change in the nature of economic fluctuations.

Panels a and b in Figure 3 compare the business cycle of China with that of the US. While aggregate employment is volatile and highly procyclical in the United States, aggregate employment is acyclical and relatively smooth in China.[6]

Interestingly, the aggregate fluctuations in employment is systematically associated with structural change and movements in and out of agriculture. Consider panel c in Figure 3. Until 1960, NBER recessions were associated with a slowdown and reversal of the process of structural change in the US. Namely, the employment share of agriculture fell in booms and increased in downturns. The cyclical pattern of employment in agriculture is dampened in the later part of the sample and ceases to be visible after 1960. Note that China today looks similar to the US of the early days. Panel d in Figure 3 shows that structural change – measured as the decline in agricultural employment – accelerates during periods of high growth and slows down or halts during periods of low growth in China. This pattern is even more pronounced when considering the cyclical fluctuations in employment in agriculture versus non-agriculture.

Panel b of Figure 4 documents that agricultural employment is volatile and strongly countercyclical in China. There is no such pattern in the US in the same period: there, the correlation is positive rather than negative. Interestingly, the cyclical pattern of nonagricultural employment is very similar in the US and in China: Panels c and d of Figure 4 shows that nonagricultural

[6] Figure 12 in the appendix shows how the volatility of employment relative to the volatility of output in the US has increased over time. Before 1980, employment was significantly less volatile than output, while after 2000 employment has been more volatile than output.

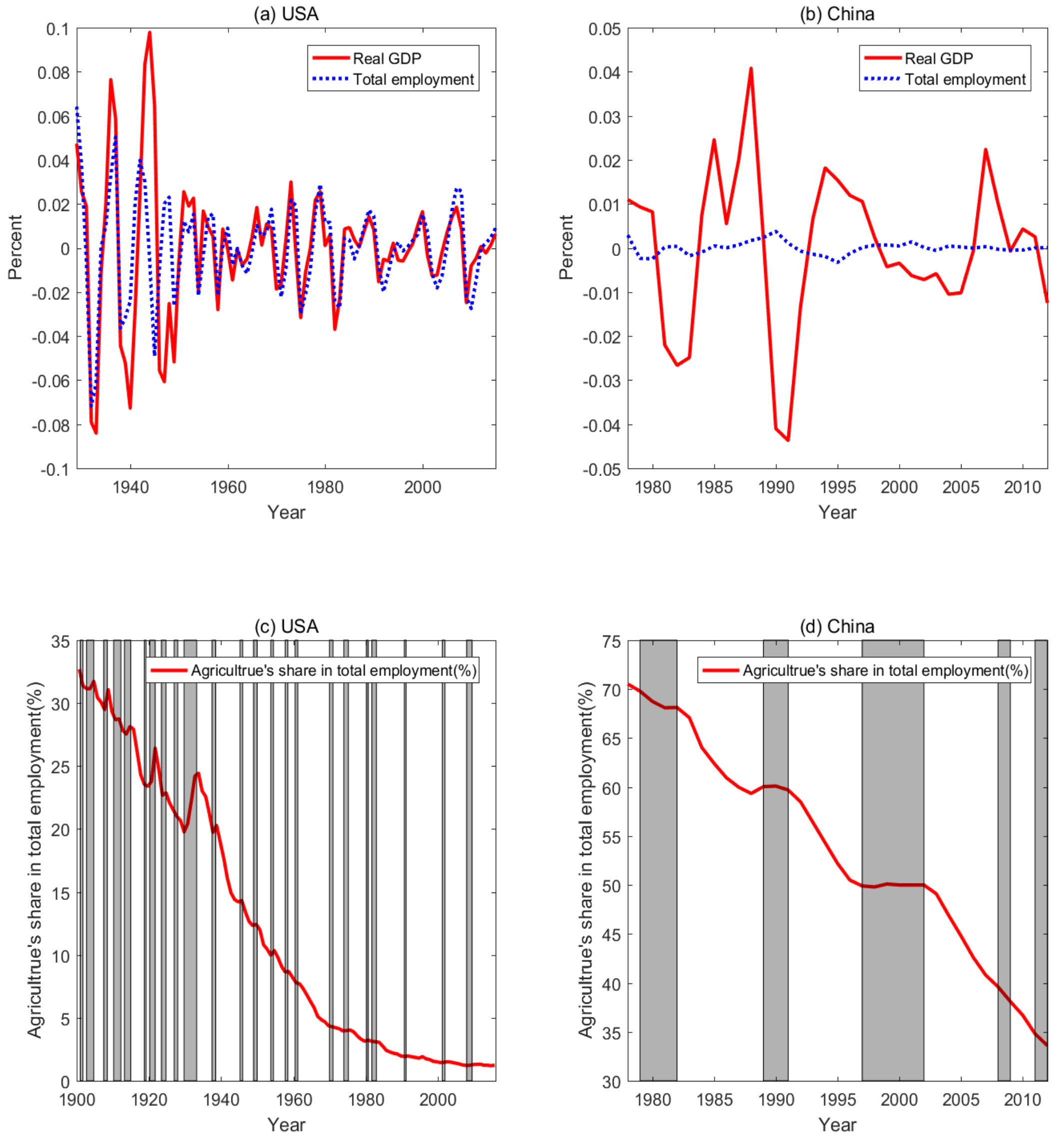



Figure 3: Panel a and b plot the volatility of total employment and real GDP in the US (1929-2015) and China (1978-2012). The figure shows the time evolution in the US (left panel) and China (right panel) of HP-filtered employment and GDP. The HP-filtered use smoothing parameter 6.25 ((Ravn and Uhlig 2002)). The aggregate employment data in China is from the Statistic Year Book by the China National Bureau of Statistics, Table "Number of employed persons at year-end by three strata of industry". We incorporate a correction proposed by (Holz 2006). The correction takes care of the reclassification of employed workers that was made by the NBS in 1990. Panel c and d plot the agriculture's share in total employment over the business cycles. The left panel plots the farm employment share over the business cycle in the US. Grey ranges denote period classified as recessions by the NBER. The right panel plots the agriculture employment share over the business cycle. Grey ranges denote recessions of the Chinese economy, where the recession is defined as the period of which the real GDP growth rate is below 9.7 percent (the average real GDP growth rate in China during 1978-2012)

employment is highly procyclical and roughly as volatile as GDP both in China and the US. It follow that agricultural and nonagricultural employment are strongly negatively correlated in China.

The facts documented above are also characteristic of a set of countries.[7] First, the upper left panel in Figure 5 shows that the volatility of employment relative to GDP is weakly negatively correlated with the employment share of agriculture, consistent with the US time series evidence. The raw cross-country correlation is negative but not statistically significant. From a cross-country perspective the volatility of employment in China appears particularly low. Second, the lower left panel in Figure 5 shows that the correlation between aggregate employment and GDP declines strongly with the employment share of agriculture, being large and positive for highly industrialized countries such as the US and negative for countries with large agricultural sectors. This is consistent with the US-China pattern discussed above.

The right upper panel in Figure 5 shows that the time-series correlation of the HP-filtered employment in agriculture and nonagriculture is positive (on average) for countries with a small agricultural sector like the US and negative for countries with a large agricultural sector like China.

Finally, we study the dynamics of the productivity gap, i.e., the average labor productivity in nonagriculture relative to agriculture. Recall that during the process of structural change, the productivity gap has fallen (see panel b of Figures 1-2). Thus, employment in agriculture fell while productivity in agriculture increased. The lower right panel in Figure 5 shows that the same fact holds true at business-cycle frequencies. Namely, in countries with large agricultural sectors, the labor productivity gap is negatively correlated with employment in nonagriculture, while this correlation is close to zero in countries with a small agricultural sector. For instance the correlation is $-0.54$ for China. We conclude that relative productivity and relative employment (in agriculture) move in opposite directions. This happens both during the process of structural change and over the business cycle in countries that are undergoing structural change away from agriculture.

In summary, the characteristics of the business cycles change systematically across different stages of the process of structural change. In countries with a large agricultural sector, we

[7] We use sector-level employment data from the International Labor Organization (ILO). The appendix describes how the data set is constructed.

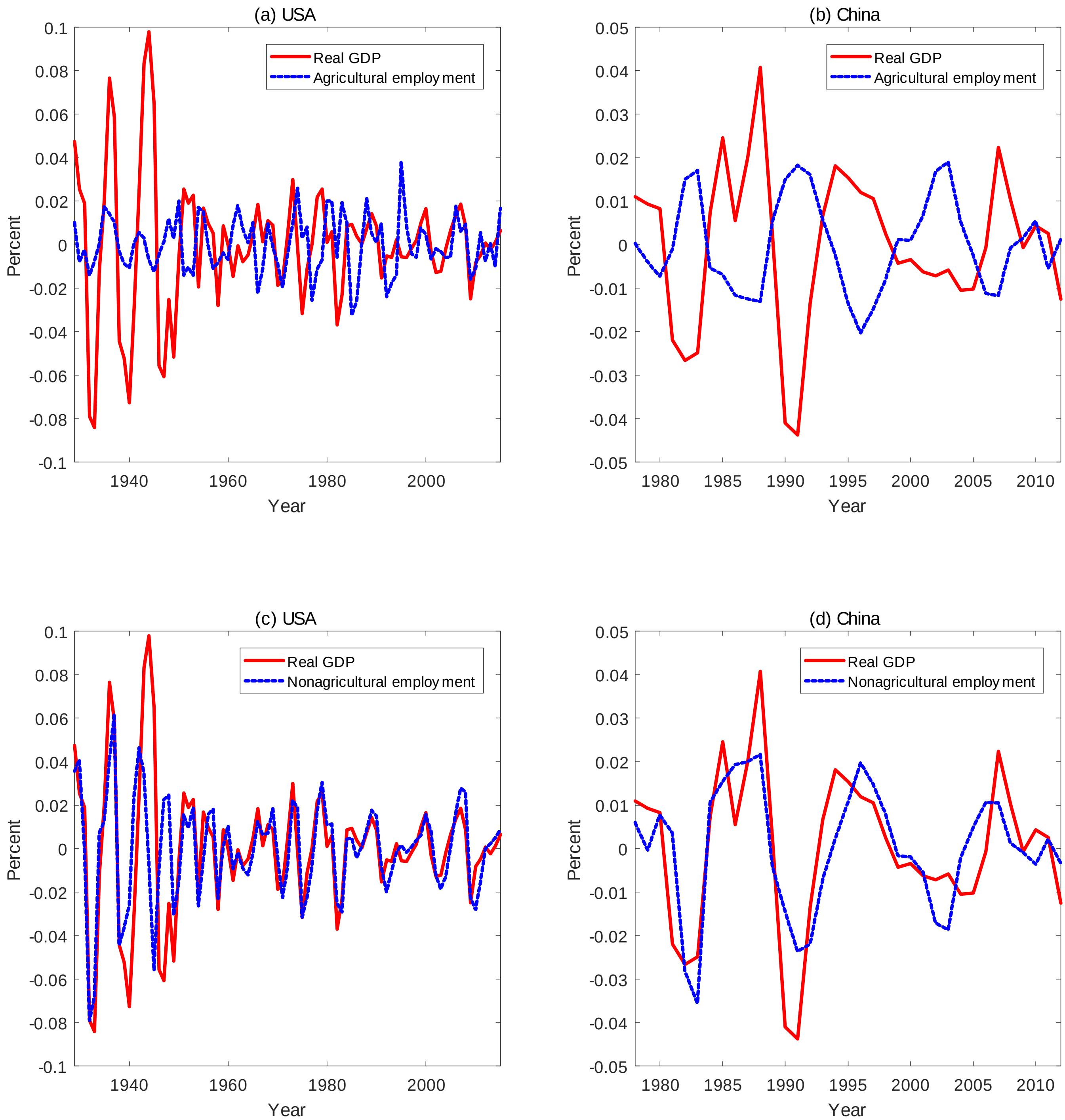



Figure 4: Panel a and b plot the HP-filtered log agricultural employment vs HP-filtered log real GDP in the US 1929-2015 and in China 1978-2012. Panel c and d plot the HP-filtered log nonagricultural employment vs HP-filtered log real GDP in the US and in China. We use a smoothing parameter 6.25 for the HP filter (Ravn and Uhlig 2002). Source: The US employment by sectors is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. The sectoral employment data in China is from the Statistic Year Book by the China National Bureau of Statistics, Table "Number of employed persons at year-end by three strata of industry". The number is calculated based on the households survey on both urban and rural households in China. The nonagriculture employment is the sum of both employment in the secondary industry and the employment in the tertiary industry. We incorporate a correction proposed by (Holz 2006). The correction takes care of the reclassification of employed workers that was made by the NBS in 1990.

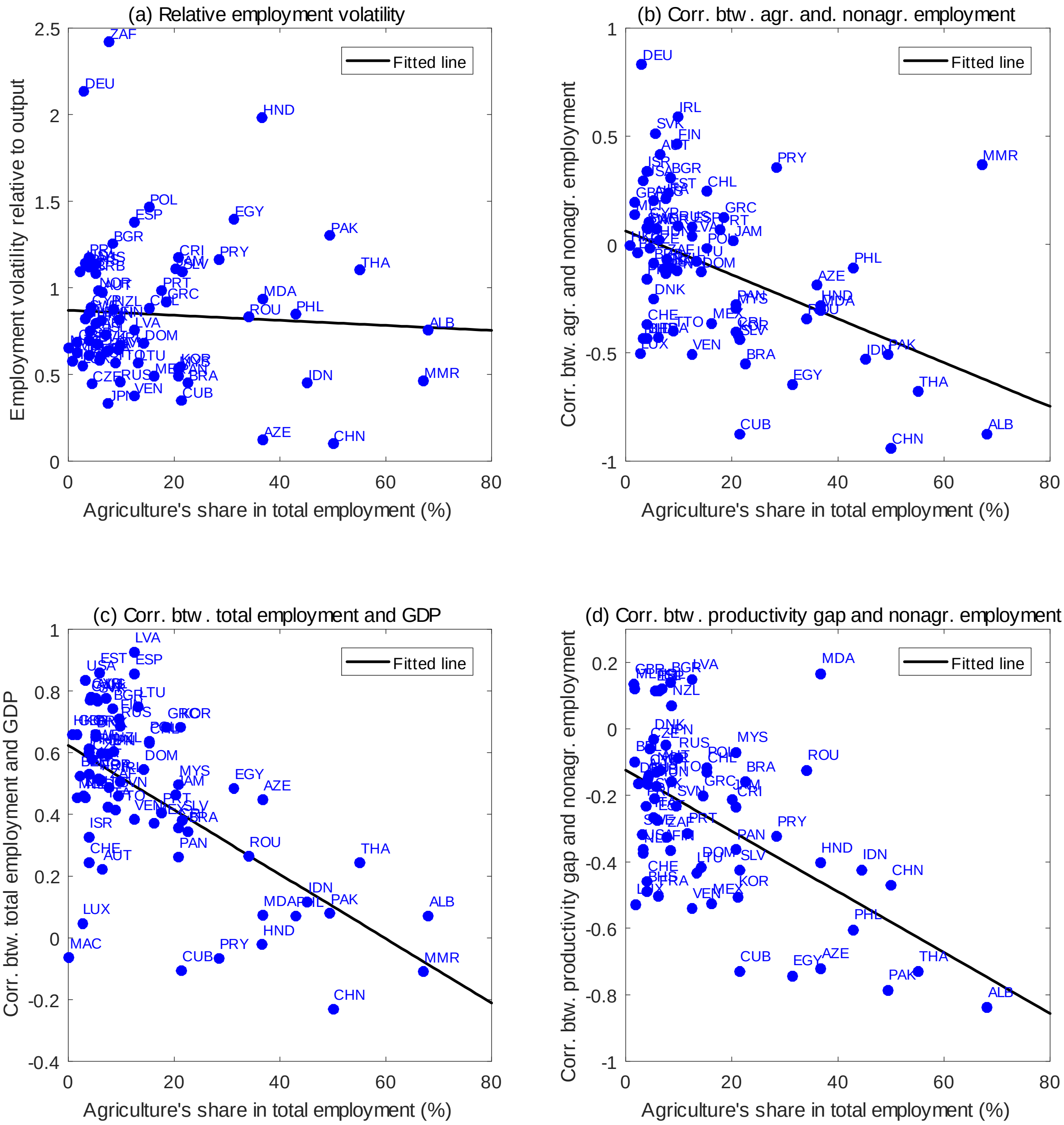



Figure 5: The figure is a cross-country scatter plot of several business cycle statistics. Panel a plots the relative volatility of employment in a sample of 68 countries, where the relative volatility is measured by the standard deviation of HP-filtered log total employment divided by the standard deviation of the HP-filtered log real output. Panel b plots the time series correlation of HP-filtered log nonagricultural employment and HP-filtered log agricultural employment in a sample of 67 countries. Panel c shows the correlation between the HP-filtered log total employment and HP-filtered log real GDP in a sample of 68 countries. Panel d plots the time series correlation of the HP-filtered log productivity gap and the HP-filtered log nonagricultural employment in a sample of 63 countries. We use a smoothing parameter 6.25 for the HP filter (Ravn and Uhlig 2002). The x-axis denotes the mean agriculture's share in total employment over the sample period for each country. Sample period covers 1970-2015 and some countries have fewer observations. We keep the countries that have at least more than 15 years of consecutive observations in order to calculate the business cycle statistics.

observe:

1. Aggregate employment has a low (and sometimes negative) correlation with GDP;

2. The cyclical employment in agriculture is strongly countercyclical and more pronounced;

3. The labor productivity gap is negatively correlated with employment in nonagriculture.

The third point is important as it suggests that the mechanisms driving structural change might be related to those driving the business cycle fluctuations. In economies with a large agriculture, recessions are times of slowdown and even reversal of structural change. People stop leaving or even move back to rural areas and recessions have a sullying effect on the productivity of agriculture. In fully modernized economies, farms live an almost an independent life and workers move in and out employment in the urban sector.

# 3 A Model of Business Cycle with Structural Change

In this section, we present a dynamic general equilibrium model that describes the process of growth and structural change in an economy with a declining agricultural sector. We show that under appropriate restrictions on the cross-sectoral elasticities and on the sectoral TFP growth rates, the model predicts trajectories of structural change, and modernization of agriculture that resemble those observed in the data. We first derive some characterization results. Then, in the following section, we estimate the key structural parameters of the deterministic version of the model using data for China. We show that the model matches accurately the process of structural change in China. Finally, we introduce uncertainty and productivity shocks, and show that the stochastic version of the model with parameters calibrated to the structural change of China is consistent with salient business cycle features of economies undergoing a process of structural change. As documented in the previous section, these are different from those of fully industrialized economies.

## 3.1 Environment

### 3.1.1 Production

The consumption good, assumed to be the numeraire, is a CES aggregate of two goods,

$$Y = F\left(Y^G, Y^M\right) = \left[\gamma\left(Y^G\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(Y^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1}$$

We label the sectors producing the two goods *agriculture* (superscript G) and *nonagriculture* (superscript M, as in "manufacturing"), respectively. We denote by $\varepsilon > 0$ the elasticity of substitution between the two goods.[8]

The technology of nonagriculture is described by the following Cobb Douglas production function

$$Y^M = \left(K^M\right)^{1-\alpha} \times \left(Z^M H^M\right)^{\alpha}, \tag{2}$$

where $H^M = hN^M$ is the labor input. $N^M$ denotes the number of workers and $h$ denotes the number of hours worked by each of them. $K^M$ denotes capital and $Z^M$ is a productivity parameter (henceforth, TFP).[9]

Agriculture is a CES aggregate of two subsectors, modern agriculture (superscript AM) and traditional agriculture (superscript S, as in "subsistence"), which produce imperfect substitutes with an elasticity of substitution $\omega > 0$. More formally,

$$Y^G = \left[\varsigma\left(Y^{AM}\right)^{\frac{\omega-1}{\omega}} + (1-\varsigma)\left(Y^S\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{3}$$

where $\varsigma \in (0,1)$. Modern agriculture uses a Cobb-Douglas technology with capital and labor (in the quantitative section we will allow for land in agriculture):

$$Y^{AM} = \left(K^{AM}\right)^{1-\beta}\left(Z^{AM} H^{AM}\right)^{\beta}. \tag{4}$$

[8] The technology parameters $\gamma$ and $\varepsilon$ can alternatively be interpreted as preference parameters, reflecting the relative weight and the elasticity of substitution between goods produced by the agricultural and nonagricultural sector. The same interpretation can be given to the parameters $\zeta$ and $\omega$ in Equation (3) below.

[9] With Cobb Douglas technology, the distinction between different types of technical progress is immaterial. Thus, referring to $Z^M$ (and later to $Z^A$) as total factor productivity (TFP) is without loss of generality.

Traditional agriculture does not use any capital:

$$Y^S = Z^S H^S. \tag{5}$$

Note that the presence of a labor-intensive traditional sector implies a variable elasticity of substitution between capital and labor in agriculture. When $\omega > 1$, the elasticity of substitution is larger than unity and declines as the economy develops. As $K^{AM}$ grows large, the elasticity of substitution approaches unity.

We assume the TFPs $Z^M$, $Z^{AM}$, and $Z^S$ to grow at constant exponential rates $g^M$, $g^{AM}$, and $g^S$, respectively. All goods are produced competitively. Both capital and labor are perfectly mobile across sectors. Capital depreciates at the rate $\delta$.

### 3.1.2 Households

Agent's preferences are described by a logarithmic utility function.

$$U = \int_0^\infty (\theta \log c + (1-\theta)\log(1-h)) \times e^{-(\rho-n)t} dt. \tag{6}$$

Here, $c \equiv C/N$ denotes the consumption per capita, $1-h$ is leisure, and $\rho$ is the time discount rate. The population grows at the exogenous rate $n < \rho$. The analysis can be extended to a general CRRA utility function. In the analytical section, for simplicity, we assume an inelastic labor supply, set $\theta = 1$ and ignore leisure altogether (i.e., we set $H^i = N^i$). We introduce an endogenous labor-leisure choice in the quantitative analysis below where we estimate the model and study economic fluctuations (when we introduce uncertainty, (6) is replaced by an expected utility function with unit relative risk aversion.). We suppress time indexes when it is not a source of confusion.

The representative household maximizes expected utility subject to a set of period budget constraints $Nc + \dot{K} = wN + RK + Tr$, where $K = K^M + K^{AM}$ and $N = N^M + N^{AM} + N^S$ denote the aggregate capital stock and number of workers, respectively. $w$ denotes the after-tax wage that is equalized across sectors in equilibrium; $R$ denotes the equilibrium (gross) interest rate.

Since in the data we see persistent labor wage differences across agriculture and nonagriculture, we introduce an exogenous wedge by assuming that the government taxes wages in nonagriculture at the rate $\tau$.[10] The government runs a balanced budget each period and rebates the tax proceeds to the representative household as lump-sum transfers, denoted by $Tr$. Thus, $Tr = \tau W^M H^M$, where $W^M$ denotes the nonagriculture pre-tax wage. In equilibrium, $W^{AM} = W^S = (1-\tau) W^M = W$. Note that the wedge prevents the equalization of the marginal product of labor across sectors, increasing employment in agriculture and reducing the equilibrium after-tax wage $W$. The wedge is a stand-in for frictions leading to rural overpopulation. Endogenizing such frictions is left to future research.

## 3.2 Competitive Equilibrium

Since the wedge $\tau$ is the only distortion, we can characterize the recursive competitive equilibrium as the solution to the constrained maximization of a benevolent social planner problem. More formally, the planner maximizes (6) subject to the resource constraint

$$\dot{K} = F\left(Y^G, Y^M\right) - \delta K - C - \tau \bar{W} N^M + Tr, \tag{7}$$

and to the technological constraints implied by equations (1)–(5). Here, $\bar{W}$ denotes the marginal product of labor in manufacturing, i.e., $\bar{W} = W^M$. The planner takes $\bar{W}$ as parametric when calculating the first-order conditions of the maximization problem. Then, she sets $Tr = \tau \bar{W} H^M$. This procedure makes the solution of the social planner problem identical to the distorted competitive equilibrium allocation.

It is useful to introduce some useful normalizations.

**Notation 1** *Define:*

$$\begin{aligned}
c &\equiv \frac{C}{N},\ \chi \equiv \frac{K}{N}, \\
\kappa &\equiv \frac{K^M}{K},\ \nu^M \equiv \frac{N^M}{N},\ \nu^{AM} \equiv \frac{N^{AM}}{N},\ \nu^S \equiv \frac{N^S}{N} \\
\upsilon &\equiv \frac{\varsigma\left(Y^{AM}\right)^{\frac{\omega-1}{\omega}}}{\varsigma\left(Y^{AM}\right)^{\frac{\omega-1}{\omega}} + (1-\varsigma)\left(Y^S\right)^{\frac{\omega-1}{\omega}}}.
\end{aligned}$$

[10] This wedge has no effect on our analytical result but it is relevant in the quantitative analysis.

$\chi$ is the aggregate capital-labor ratio, the key endogenous state variable of the economy. $\kappa$ is the share of capital used in nonagriculture. Since the traditional sector does not use capital, $\kappa$ yields the split of the capital stock between nonagriculture and modern agriculture. $\nu^i \equiv N^i/N$ is the employment share in sector $i \in \{AM, M, S\}$. $\upsilon$ measures the GDP share of modern agriculture in total agriculture. We also introduce the following assumptions to focus on the empirically relevant case for our application.

**Assumption 1** *We assume:* $\varepsilon > 1$, $\omega > 1$, $\beta > \alpha$, *and* $g^M \geq g^{AM} \geq g^S$.

We characterize equilibrium in two stages. First, we solve the static problem defined the maximized current output per capita subject to the wedge $\tau$ and a given aggregate stock of capital per worker $\chi$ and the TFP levels. Then, we solve the dynamic equilibrium involving capital accumulation and technical progress. Given $\chi$, the competitive equilibrium maximizes the distorted output per worker $y$

$$y(\chi) = \max_{\{\kappa,\nu^S,\nu^{AM},\nu^M\}} f\left(y^G\left(\kappa,\nu^S,\nu^{AM},\nu^M,\chi\right), y^M\left(\kappa,\nu^S,\nu^{AM},\nu^M,\chi\right)\right) - \tau\bar{W}\nu^M + Tr. \quad (8)$$

subject to the technology constraint

$$\begin{aligned} & f\left(y^G\left(\kappa,\nu^S,\nu^{AM},\nu^M\right), y^M\left(\kappa,\nu^S,\nu^{AM},\nu^M\right)\right) \quad (9) \\ = & \left[\begin{array}{c} \gamma\left(\left(\begin{array}{c} \varsigma\left(\left(Z^{AM}\right)^{\alpha}\times\left((1-\kappa)\chi\right)^{1-\alpha}\times\left(\nu^{AM}\right)^{\alpha}\right)^{\frac{\omega-1}{\omega}} \\ +(1-\varsigma)\left(Z^S\nu^S\right)^{\frac{\omega-1}{\omega}} \end{array}\right)^{\frac{\omega}{\omega-1}}\right)^{\frac{\varepsilon-1}{\varepsilon}} \\ \\ +(1-\gamma)\left(\left(Z^M\right)^{\alpha}\times(\kappa\chi)^{1-\alpha}\times\left(\nu^M\right)^{\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} \end{array}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \end{aligned}$$

and the resource constraints $\kappa \in [0,1]$ and $\nu^M + \nu^{AM} + \nu^S = 1$.

Conditional on the wedge $\tau$, production efficiency requires the equalization across sectors of the marginal product of labor and of the marginal product capital.[11] The equalization of the marginal product of capital in modern agriculture and nonagriculture yields, after rearranging terms:

$$\frac{1-\kappa}{\kappa}=\frac{1-\beta}{1-\alpha}\frac{\gamma}{1-\gamma}\left(\frac{y^G}{y^M}\right)^{\frac{\varepsilon-1}{\varepsilon}}\upsilon \tag{10}$$

The equalization of the marginal product of labor in modern agriculture and nonagriculture, and in traditional agriculture and nonagriculture yield, respectively,

$$\nu^{AM} = \frac{1}{1-\tau}\frac{1-\alpha}{1-\beta}\frac{\beta}{\alpha}\frac{1-\kappa}{\kappa}\nu^M \tag{11}$$

$$\nu^S = \frac{1}{\beta}\frac{1-\upsilon}{\upsilon}\nu^{AM} \tag{12}$$

Combining (10), (11), and (12) with the resource constraint on labor, and solving for $\nu^M$, yields

$$\nu^M=\left(1+\frac{1}{1-\tau}\frac{1-\kappa}{\kappa}\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\left(1+\frac{1}{\beta}\frac{1-\upsilon}{\upsilon}\right)\right)^{-1}. \tag{13}$$

**Proposition 1** *Given* $\chi$ *and* $\boldsymbol{Z}=\left(Z^M,Z^{AM},Z^S\right)$, *a static competitive equilibrium is characterized by the functions* $\kappa=\kappa\left(\chi,\boldsymbol{Z}\right)$, $\upsilon=\upsilon\left(\chi,\boldsymbol{Z}\right)$ *and by the equilibrium employment shares* $\nu^M\left(\kappa\left(\chi,\boldsymbol{Z}\right),\upsilon\left(\chi,\boldsymbol{Z}\right)\right)$, $\nu^{AM}\left(\kappa\left(\chi,\boldsymbol{Z}\right),\upsilon\left(\chi,\boldsymbol{Z}\right)\right)$, *and* $\nu^S\left(\kappa\left(\chi,\boldsymbol{Z}\right),\upsilon\left(\chi,\boldsymbol{Z}\right)\right)$ *implicitly defined by Equations (10), (11), (12), (13), and by the technology (9).*

We can then write the aggregate production function under constrained productive efficiency as

$$y\left(\chi,\boldsymbol{Z}\right)=\eta\left(\kappa\left(\chi,\boldsymbol{Z}\right),\upsilon\left(\chi,\boldsymbol{Z}\right)\right)\times\left(\chi\kappa\left(\chi,\boldsymbol{Z}\right)\right)^{1-\alpha}\left(\nu^M\left(\kappa\left(\chi,\boldsymbol{Z}\right),\upsilon\left(\chi,\boldsymbol{Z}\right)\right)\right)^{\alpha},$$

[11]

$$\begin{aligned}
(1-\tau)\frac{\partial Y}{\partial Y^M}\frac{\partial Y^M}{\partial K^M} &= \frac{\partial Y}{\partial Y^G}\frac{\partial Y^G}{\partial Y^{AM}}\frac{\partial Y^{AM}}{\partial K^{AM}}\\
(1-\tau)\frac{\partial Y}{\partial Y^M}\frac{\partial Y^M}{\partial \nu^M} &= \frac{\partial Y}{\partial Y^G}\frac{\partial Y^G}{\partial Y^{AM}}\frac{\partial Y^{AM}}{\partial \nu^{AM}}\\
\frac{\partial Y}{\partial Y^G}\frac{\partial Y^G}{\partial Y^{AM}}\frac{\partial Y^{AM}}{\partial \nu^{AM}} &= \frac{\partial Y}{\partial Y^G}\frac{\partial Y^G}{\partial Y^S}\frac{\partial Y^S}{\partial \nu^S}
\end{aligned}$$

where

$$\eta\left(\kappa\left(\chi,\boldsymbol{Z}\right),\upsilon\left(\chi,\boldsymbol{Z}\right)\right)=\left(1-\gamma\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(1+\frac{1-\alpha}{1-\beta}\frac{1-\kappa\left(\chi,\boldsymbol{Z}\right)}{\kappa\left(\chi,\boldsymbol{Z}\right)}\frac{1}{\upsilon\left(\chi,\boldsymbol{Z}\right)}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{14}$$

### 3.2.1 Static Equilibrium for $\omega$ close to $\varepsilon$

The properties of the functions $\kappa$ and $\upsilon$ are in general involved due to nonlinearities. In this section, we establish sharp results for economies where $\omega$ is close to $\varepsilon$. In this case, structural change implies monotone dynamics in capital accumulation at all stages of economic transition. Development is characterized by the simultaneous modernization (i.e., increasing $\upsilon$) and relative decline (i.e., increasing $\kappa$) of agriculture, two features that accord well with the empirical evidence. These dynamics arise both through capital accumulation and through differential TFP growth.

We first consider the effect of capital deepening. The following result can be established for $\omega=\varepsilon$:

$$\left.\frac{\partial\ln\kappa\left(\chi,\boldsymbol{Z}\right)}{\partial\ln\chi}\right|_{\omega=\varepsilon}=\frac{\left(\varepsilon-1\right)\left(\beta-\alpha\right)\left(1-\kappa\right)}{1+\left(\varepsilon-1\right)\left(\left(\beta-\alpha\right)\left(\kappa-\nu^{M}\right)+\nu^{S}\left(1-\beta\right)\right)}>0 \tag{15}$$

The proof, which is the generalization of a result in (Acemoglu and Guerrieri 2008), is in the appendix. Note, that $\kappa-\nu^{M}>0$, since nonagriculture is the capital-intensive sector. Note also that the assumptions that $\varepsilon>1$ and $\beta>\alpha$ are key to sign this derivative. The following lemma, then, follows.

**Lemma 1** *Suppose $\beta>\alpha$, and $\varepsilon>1$. Then, there exists $\bar{x}>0$ such that, if $\|\omega-\varepsilon\|<\bar{x}$, then, both $\kappa\left(\chi,\boldsymbol{Z}\right)$ and $\upsilon\left(\chi,\boldsymbol{Z}\right)$ are monotone increasing in $\chi$. Moreover, $\nu^{M}$, $\nu^{M}/\nu^{AM}$, and $\nu^{AM}/\nu^{S}$ are monotone increasing in $\chi$ while $\nu^{S}$ is monotone decreasing in $\chi$.*

The result in the Lemma follows from (32) and the FOCs of the problem. Since $\kappa$ increases in $\chi$, then Equation (11) implies that the ratio $\nu^{M}/\nu^{AM}$ also increases with $\chi$. Moreover, the capital labor ratio in modern agriculture must be proportional to that in nonagriculture, i.e.,

$$\frac{\left(1-\kappa\right)\chi}{\nu^{AM}}=\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{\kappa\chi}{\nu^{M}}. \tag{16}$$

Thus, as the economy becomes more capital rich, both nonagriculture and modern agriculture undergo capital deepening. The FOC (12) then implies that the ratio $\nu^{AM}/\nu^{S}$ and $\upsilon$ must also increase with $\chi$.[12] In turn, $\nu^S$ must be falling and $\nu^M$ increasing. Thus, agriculture undergo modernization: both the employment share $(\nu^{AM}/\nu^S)$ and the the output share $(\upsilon)$ of modern agriculture increase with $\chi$ as shares of total agriculture.

Next, we study the effect of TFPs on the allocation of capital $(\kappa)$ and the modernization of agriculture $(\upsilon)$. First, it is straightforward to see that a proportional increase in all TFP levels does not affect $\kappa$ and $\upsilon$, i.e., $\kappa(\chi, \boldsymbol{Z}) = \kappa(\chi, \lambda\boldsymbol{Z})$ and $\upsilon(\chi, \boldsymbol{Z}) = \upsilon(\chi, \lambda\boldsymbol{Z})$ for any $\lambda > 0$. In fact, what matters are the relative TFP in the three sector. With some slight abuse of notation, let us write $\kappa = \kappa\left(\chi, z^{AM}, z^S\right)$ where $z^{AM} \equiv \left(Z^{AM}\right)^\beta / \left(Z^M\right)^\alpha$ and $z^S \equiv Z^S / \left(Z^M\right)^\alpha$.

**Lemma 2** *Suppose $\beta > \alpha$, and $\varepsilon > 1$. Then, $\kappa\left(\chi, z^{AM}, z^S\right)$ is decreasing in $z^{AM}$ and increasing in $z^S$, whereas $\upsilon\left(\chi, z^{AM}, z^S\right)$ is increasing in $z^{AM}$ and decreasing in $z^S$. Moreover, $\nu^{AM}$, $\nu^{AM}/\nu^S$ and $\nu^{AM}/\nu^M$ are increasing in $z^{AM}$ and decreasing in $z^S$.*

Lemma 2 establishes that higher TFP in nonagriculture relative to modern agriculture (while holding constant $Z^S$) causes a reallocation of both capital and labor towards nonagriculture. Likewise, higher TFP in modern relative to traditional agriculture (while holding constant $Z^M$) causes the modernization of agriculture.

## 3.3 Dynamic Equilibrium

In this section, we characterize the dynamic equilibrium. We continue to exploit the equivalence between the planning solution and the competitive equilibrium. Thus, we can write:

$$\max_{\left[c_t, \chi_t, \kappa_t, \nu_t^M, \nu_t^{AM}, \nu_t^S\right]_{t=0}^{\infty}} U = \int_0^\infty \log(c) \times e^{-(\rho-n)t} dt$$

subject to the resource constraint

$$\dot{\chi} = f\left(y^G\left(\kappa, \nu^M, \nu^{AM}, \nu^S, \chi\right), y^M\left(\kappa, \nu^M, \nu^{AM}, \nu^S, \chi\right)\right) - \left(\delta + n + g^M\right)\chi - c,$$

[12] To see this, rewrite (12) as

$$\frac{\nu^{AM}}{\nu^S} = \left(\frac{\varsigma}{1-\varsigma}\beta\right)^\varepsilon \left(\frac{\left(Z^{AM}\right)^\beta}{Z^S}\right)^{\varepsilon-1} \left(\frac{(1-\kappa)\chi}{\nu^{AM}}\right)^{(1-\beta)(\varepsilon-1)}.$$

This expression shows that the ratio $\frac{\nu^{AM}}{\nu^S}$ will increase in $\chi$ if and only if the term $\frac{(1-\kappa)\chi}{\nu^{AM}}$ increases in $\chi$.

where $f\left(y^G, y^M\right)$ is given by (9), and to the exogenous law of motion of TFPs, $\dot{Z}_t^M/Z_t^M = g^M$, $\dot{Z}_t^{AM}/Z_t^{AM} = g^{AM}$, and $\dot{Z}_t^S/Z_t^S = g^S$. The problem is subject to a vector of initial conditions $(\chi_0, \boldsymbol{Z}_0) = \left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right)$.

It is useful to break down the analysis into two steps. First, we solve the static problem, involving constrained productive efficiency, as discussed in the previous section. This yields a set policy functions $\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t), \nu^S(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t)), \nu^{AM}(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t))$, $\nu^S(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t))$. Rearranging the set of static equilibrium conditions allows us to eliminate $\nu_t^{AM}$ and $\nu_t^S$ from the maximization problem. The present-value Hamiltonian can then be written as

$$H(c_t, \chi_t, \boldsymbol{Z}_t, \xi_t) = e^{-(\rho-n)t}\log(c_t) + \xi_t \left( \begin{array}{c} \eta(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t)) \times \left(\frac{\chi_t\kappa(\chi_t, \boldsymbol{Z}_t)}{\nu^M(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t))}\right)^{1-\alpha} \\ \times \left(\nu^M(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t))\right) - (\delta+n)\chi_t - c_t \end{array} \right),$$

where $\xi_t$ is a dynamic Lagrange multiplier and $\boldsymbol{Z}_t = \left(Z_t^M, Z_t^{AM}, Z_t^S\right)$. The following proposition characterizes the equilibrium:

**Proposition 2** *The dynamic competitive equilibrium is characterized by the following system of ordinary differential equations*

$$\begin{aligned} \frac{\dot{c}_t}{c_t} &= (\eta(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t)))^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\left(\frac{\kappa(\chi_t, \boldsymbol{Z}_t)\chi_t}{Z_t^M\nu^M(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t))}\right)^{-\alpha} - \delta - \rho \quad (17) \\ \dot{\chi}_t &= \eta(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t)) \times (\chi_t\kappa(\chi_t, \boldsymbol{Z}_t))^{1-\alpha}\left(Z_t^M\nu^M(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t))\right)^{\alpha} - (\delta+n)\chi_t \quad (18) \\ \frac{Z_t^M}{Z_t^M} &= g^M, \; \frac{Z_t^{AM}}{Z_t^{AM}} = g^{AM}, \; \frac{Z_t^S}{Z_t^S} = g^S \end{aligned}$$

*where $\eta(\kappa, \upsilon)$ is given by (14), $\nu^M(\kappa, \upsilon)$ satisfies (13), and $\kappa(\chi_t, \boldsymbol{Z}_t)$ and $\upsilon(\chi_t, \boldsymbol{Z}_t)$ are the static equilibrium policy functions. The solution is subject to a vector of initial conditions $(\chi_0, \boldsymbol{Z}_0) = \left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right)$ and a transversality condition (see the appendix)*

Eq. (17) is a standard Euler equation for consumption. For constant TFPs, the growth rate of consumption is decreasing in $\chi$ because the aggregate production function exhibit decreasing returns to capital. However, technical change sustains the marginal product of capital. As we show below, in an asymptotic balanced growth path these two opposite forces exactly offset each other leading to a constant growth rate of consumption per capita.

It is possible to write the set of static and dynamic conditions in terms of an autonomous system of differential equations. To this aim, we differentiate with respect to time the set of static equilibrium (10), (11), (12), and (13). After rearranging terms, we obtain:

$$\frac{\dot{\kappa}_t}{\kappa_t} = (1-\kappa_t)\frac{\left(\begin{array}{c}\left(\alpha g^M - \beta g^{AM} + (\beta-\alpha)\frac{\dot{\chi}_t}{\chi_t}\right) + \\ \left(\frac{1}{\omega-1} - \frac{(\beta-\alpha)\left(1-\nu^M(\kappa_t,\upsilon_t)\right)}{1-\upsilon_t(1-\beta)}\right)\frac{\dot{\upsilon}_t}{\upsilon_t}\end{array}\right)}{\frac{1}{\varepsilon-1} + (\beta-\alpha)\left(\kappa_t - \nu^M(\kappa_t,\upsilon_t)\right)}, \tag{19}$$

$$\frac{\dot{\upsilon}_t}{\upsilon_t} = \frac{(1-\upsilon_t)\left(\beta g^{AM} - g^S + (1-\beta)\left(\frac{\dot{\chi}_t}{\chi_t} - \frac{\dot{\kappa}_t}{\kappa_t}\frac{\kappa_t-\nu^M(\kappa_t,\upsilon_t)}{1-\kappa_t}\right)\right)}{\frac{1}{\omega-1} + \frac{(1-\upsilon_t)(1-\beta)(1-\nu^M(\kappa_t,\upsilon_t))}{1-\upsilon_t(1-\beta)}}. \tag{20}$$

This dynamic system is defined up to a pair of initial conditions: $\kappa_0 = \kappa(\chi_0, \boldsymbol{Z}_0)$ and $\upsilon_0 = \upsilon(\chi_0, \boldsymbol{Z}_0)$ consistent with the static equilibrium conditions at time zero.

**Lemma 3** *The dynamic competitive equilibrium is fully characterized by the solution to the system of ordinary differential equations (17)-(18)-(19)-(20) and the exogenous law of motion $\dot{Z}_t^M/Z_t^M = g^M$, after setting $\kappa(\chi_t, \boldsymbol{Z}_t) = \kappa_t$ and $\upsilon(\chi_t, \boldsymbol{Z}_t) = \upsilon_t$ for $t > 0$, $\kappa(\chi_0, \boldsymbol{Z}_0) = \kappa\left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right) \equiv \kappa_0$ and $(\chi_0, \boldsymbol{Z}_0) = \upsilon\left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right) \equiv \upsilon_0$.*

Note that Equations (19)-(20) allow us to eliminate $Z_t^{AM}$ and $Z_t^S$ from the dynamic system, while only retaining their initial levels and their growth rates. In other words, $\kappa_0 = \kappa\left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right)$ and $\upsilon_0 = \upsilon\left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right)$ are sufficient statistics. When $\kappa_0$ and $\upsilon_0$ are at the static equilibrium level at time zero, Equations (19)-(20) guarantee that they will also be in equilibrium in all future periods.

This characterization allows us to prove conditions under which the economy converges to an asymptotic balanced growth path (ABGP) where the agriculture is fully modernized and the share of nonagriculture in total GDP is unity.

**Proposition 3** *Let $k^M = \kappa\chi$ and $k^{AM} = (1-\kappa)\chi$. Then, there exists an Asymptotic Balanced*

*Growth Path (ABGP) such that*

$$\begin{aligned}
\frac{\dot{c}_t}{c_t} &= \frac{\dot{\chi}_t}{\chi_t} = \frac{\dot{k}^M}{k^M} = g_M; \\
\kappa &= \nu^M = 1;\ \frac{\dot{\kappa}_t}{\kappa_t} = \frac{\dot{\upsilon}_t}{\upsilon_t} = 0; \\
\frac{\dot{k}^M}{k^M} &= g^M,\ \frac{\dot{k}^{AM}}{k^{AM}} = g^M - (\varepsilon - 1)\,\beta\left(g^M - g^{AM}\right); \\
\frac{\dot{N}^M}{N^M} &= n,\ \frac{\dot{N}^{AM}}{N^{AM}} = n - (\varepsilon - 1)\,\beta\left(g^M - g^{AM}\right); \\
\frac{\dot{N}^S}{N^S} &= \frac{\dot{N}^{AM}}{N^{AM}} - (\omega - 1)\left[\left(g^{AM} - g^S\right) + (1-\beta)\left(g^M - g^{AM}\right)\right].
\end{aligned}$$

*Along the ABGP*

$$\left(\frac{c}{\chi}\right)^* = \left(\frac{g^M + \delta + \rho}{1-\alpha}\right) - \left(g^M + \delta + n\right), \tag{21}$$

$$\left(\frac{\chi}{Z^M}\right)^* = \left(\frac{(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}\,(1-\alpha)}{g^M + \delta + \rho}\right)^{\frac{1}{\alpha}}. \tag{22}$$

*If $\varepsilon > 1$, $\omega > 1$, $\beta > \alpha$, and $g^M \geq g^{AM} \geq g^S$, then, the ABGP is asymptotically stable. Given $\left(\chi_0/Z_0^M, \kappa_0, \upsilon_0\right)$ close to the ABGP, the economy converges to the ABGP.*

The ABGP features a vanishing GDP and employment share of agriculture in the total economy. This result is driven by two assumptions. First, the elasticity of substitution between the nonagricultural and agricultural product is larger than unity. Second, technical progress is at least as high in nonagriculture as in agriculture. Note that as long as $\varepsilon$ is not too large, $\beta$ is not too large and the TFP growth gap between nonagriculture and modern agriculture is not too high, capital accumulation remains positive in modern agriculture in the ABGP (i.e., $k^{AM}$ grows at a positive rate), although the share of capital that goes to agriculture tends to zero (i.e., $\kappa \to 1$). In addition, the ABGP features modernization of agriculture: Traditional agriculture vanishes as both GDP and employment share of total agriculture. This is due to the combination of a high elasticity of substitution ($\omega > 1$) and (weakly) faster technical progress in modern agriculture than in nonagriculture.

Our theory bears predictions about the labor income shares and the labor productivity gap.

To highlight them, we move from the planner's allocation to its decentralization through prices. Denote by $LIS^j \equiv W^j H^j / P^j Y^j$ for $j \in \{G, M\}$ the sectoral labor income share. The labor share in nonagriculture ($LIS^M$) is constant, owing to the Cobb-Douglas production function. The labor share in agriculture ($LIS^G$) declines throughout development, owing to the progressive demise of the labor-intensive traditional sector. More precisely, the labor share in agriculture declines from unity – when capital is very low and the agriculture is dominated by the traditional sector – to $\beta$ when the agriculture is fully modernized.[13] The declining labor share in agriculture is consistent with Figure 1 above. Consider, next, the sectoral (average) labor productivity $APL^j \equiv P^j Y^j / H^j$. Our theory predicts that labor productivity grows faster in agriculture than in nonagriculture as capital accumulates.[14] This is consistent with the evidence in Figures 1 and 2 above.

We summarize these properties in the following Corollary of Proposition 3 (proof in appendix).

Let $APL^j \equiv P^j Y^j / H^j$ and $LIS^j \equiv W^j H^j / P^j Y^j$ denote the average product of labor and the labor income share in sector $j \in \{G, M\}$, respectively. Then, the labor productivity gap is given by

$$\frac{APL^M}{APL^G} = \frac{1}{1-\tau}\frac{LIS^G}{LIS^M} = \frac{1}{1-\tau}\frac{\beta \upsilon + (1-\upsilon)}{\alpha}$$

Under the conditions of Proposition 3, we have

$$\begin{aligned}
\lim_{t\to\infty} \frac{LIS^G}{LIS^M} &= \frac{\beta}{\alpha} \\
\lim_{t\to\infty} \frac{APL^M}{APL^G} &= \frac{1}{1-\tau}\frac{\beta}{\alpha}.
\end{aligned}$$

[13] One could obtain a declining labor share by assuming an aggregate CES production function in agriculture.with a high elasticity of substitution between capital and labor, like in (Alvarez-Cuadrado, Long, and Poschke 2017). However, such alternative model would feature, counterfactually, an ever declining labor share that would converge to zero in the long run. In our model, like in the data, the labor share in agriculture declines but remains bounded away of zero.

[14] Note that the two predictions are two sides of the same coin. In an undistorted economy, $APL^M/APL^A = \left(LIS^M/LIS^A\right)^{-1}$ by definition. In the distorted economy, the labor tax in nonagriculture opens a wedge between the marginal product of labor in the two sectors that is reflected in the equilibrium prices. Thus, one can show that $APL^M/APL^A = \left((1-\tau) \times LIS^M/LIS^A\right)^{-1}$.

### 3.3.1 Equilibrium in the Lewis Model

A particular case for which a global characterization of the equilibrium dynamics is available is one where the output of traditional and modern agriculture are perfect substitutes (i.e., where $\omega \to \infty$), implying that labor productivity and the wage are fixed in terms of the agricultural product. This model is interesting for being close to the seminal contribution of (Lewis 1954). In addition, it illustrates why the dynamics of $\kappa$ and $\upsilon$ may be nonmonotone. Intuitively, when $\omega$ is large, labor in the traditional sector is a closed substitute of capital in modern agriculture. When capital is scarce, it is then efficient to allocate the entire stock of capital to nonagriculture while deferring the modernization of agriculture to a later stage in which capital is more abundant.

For simplicity, we abstract in this section from technical progress and set $Z^M = Z^{AM} = Z^S = 1$. Moreover, we set $\tau = 0$. Endogenous capital accumulation is then the only source of transition. We continue to assume that $\varepsilon > 1$ and $\beta > \alpha$.

The dynamic equilibrium evolves through three stages. In the first stage, (*Early Lewis* stage), capital is very scarce, all agricultural production takes place in the traditional sector ($\upsilon = 0$) and all capital is allocated to the manufacturing sector ($\kappa = 1$). In the second stage (*Advanced Lewis* stage), capital and labor are also allocated to the modern agricultural sector, while employment declines in the traditional sector. During this stage $\kappa$ declines and $\upsilon$ increases. Finally, when the labor force reserve in traditional agriculture is exhausted, the economy enters the third stage (*Neoclassical* stage). In this stage, all the agricultural production takes place in modern farms using capital ($\upsilon = 1$). Since $\varepsilon > 1$ and nonagriculture is more capital intensive than agriculture, the output share of manufacturing keeps growing. Moreover, $\kappa$ increases.

**Stage 1 (Early Lewis).** Consider an economy in which capital is very scarce. When $\chi < \underline{\chi}$, then, $\nu^{AM} = 0$, $\nu^M > 0$, $\nu^S > 0$, and $\kappa = 1$.[15] Intuitively, because capital is scarce, it is optimal to use it only in nonagriculture, where it is an essential factor, to take advantage of the high relative price of the nonagricultural good. The key equilibrium condition is the equalization of the marginal product of labor in nonagriculture and traditional agriculture. Using the implicit

[15] In particular, $\underline{\chi} = \frac{\beta(1-\alpha)}{\alpha(1-\beta)} \left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}} \frac{\Xi}{1+\Xi}$, where $\Xi \equiv \left(\frac{\alpha(1-\gamma)}{\gamma(1-\zeta)^{\frac{\varepsilon-1}{\varepsilon}}}\right)^{\varepsilon} \left(\frac{\beta(1-\alpha)}{\alpha(1-\beta)}\left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}}\right)^{(\varepsilon-1)(1-\alpha)}$.

function theorem, we can show that $\nu^M$ is an increasing function of $\chi$. More formally,

$$\frac{\nu^M}{1-\nu^M} = \left(\frac{\alpha(1-\gamma)}{\gamma(1-\zeta)^{\frac{\varepsilon-1}{\varepsilon}}}\right)^{\varepsilon} \left(\frac{\chi}{\nu^M}\right)^{(\varepsilon-1)(1-\alpha)}, \tag{23}$$

where the LHS is increasing in $\nu^M$ and the RHS is increasing in $\chi$ and decreasing in $\nu^M$. Thus, standard differentiation implies that $\partial\nu^M/\partial\chi > 0$.

The average labor productivity is higher in nonagriculture than in agriculture, reflecting the nonagriculture uses capital. More formally, the *productivity gap* is given by the inverse ratio of the labor-income shares in the two sectors,

$$\frac{\frac{P^M Y^M}{L^M}}{\frac{P^G Y^G}{L^G}} = \frac{1}{\alpha}.$$

Consider, next, the evolution of the aggregate capital-output ratio and of the factor prices in the Early Lewis stage. If the agricultural and nonagricultural goods were perfect substitutes, both the wage and the interest rate would stay constant as capital accumulates. However, for $\varepsilon < \infty$ capital accumulation triggers an increase in the relative price of agricultural goods and a growth in the real wage. Wage growth in turn causes capital deepening in the nonagricultural sector and a declining interest rate. A formal proof of these statements is in the appendix.

**Stage 2 (Advanced Lewis).** As capital accumulation progresses, the relative price of the agricultural good increases. Once capital is sufficiently abundant, (i.e., as $\chi \geq \underline{\chi}$), the relative price of agriculture is so high that it becomes optimal to put some capital in the modern agricultural sector. At this point the modernization process of agriculture starts and the economy enters the Advanced Lewis stage.

In this stage the equalization of factor returns across the two sectors implies that they have a constant capital-labor ratios. These are given by

$$k^{AM} \equiv \frac{(1-\kappa)\chi}{\nu^{AM}} = \left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}}, \; k^M \equiv \frac{\kappa\chi}{\nu} = \frac{\beta(1-\alpha)}{\alpha(1-\beta)} k^{AM}. \tag{24}$$

The share of capital that goes to nonagriculture declines over the process of development:

$$\kappa = \frac{K^M}{L^M}\frac{L^M}{L}\frac{1}{\chi} = \frac{1 + \left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}} \frac{\beta}{1-\beta}\frac{1}{\chi}}{1 + \frac{\alpha}{1-\alpha}\left(\frac{1+\Xi}{\Xi}\right)} \tag{25}$$

The optimal allocation of labor in manufacturing and modern agriculture yields

$$\nu^M = \frac{\chi + k^{AM}\frac{\beta}{1-\beta}}{k^{AM}\frac{\beta}{1-\beta}\left(\frac{1+\Xi}{\Xi}\right) + k^M}, \quad \nu^{AM} = \frac{\beta}{1-\beta}\frac{1+\Xi}{\Xi}\nu^M - \frac{\beta}{1-\beta}. \tag{26}$$

These expressions shows that employment in both manufacturing and modern agriculture increase as $\chi$ grows. Since the sectoral capital-labor ratios are constant, this also implies that capital and output in these sectors are increasing at the expense of a falling production of the traditional agriculture. Since factor prices are constant while the aggregate capital intensity in the economy is increasing, then the aggregate share of GDP accruing to capital grows while the labor share falls.

An interesting observation is that throughout this stage the expenditure share on agriculture and nonagriculture remain constant, even though $\varepsilon \neq 1$. To understand why, consider an economy without a Lewis sector. In this case, when $\varepsilon > 1$ and $\beta > \alpha$, capital accumulation would imply that the capital-intensive sector (in our case, nonagriculture) would grow faster over time. Although this implies an increase in the relative price of the agricultural product, the expenditure share on nonagricultural goods would increase over time. However, reallocation within agriculture with the decline of the Lewis sector offsets this force by increasing labor productivity in agriculture.

More formally, we can show that $\frac{P^M Y^M}{P^G Y^G} = \frac{1-\gamma}{\gamma}\left(\frac{Y^M}{Y^G}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \Psi$, where $\Psi \equiv \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{\alpha}{1-\zeta}\right)^{\varepsilon-1}\left(k^M\right)^{(1-\alpha)(\varepsilon}$ This immediately implies that the productivity gap between agriculture and nonagriculture is shrinking, since

$$\frac{\frac{P^M Y^M}{L^M}}{\frac{P^G Y^G}{L^G}} = \Psi\frac{1-\nu^M}{\nu^M},$$

and, recall, $\nu^M$ is increasing in the Advanced Lewis stage. In particular, the productivity gap (which is the inverse of the ration between the labor income share in the two sectors) declines from $1/\alpha$ to $\beta/\alpha$ in this stage, where, recall, $\beta$ is the labor income share in modern agriculture.

Finally, in the Advanced Lewis stage, the capital-output ratio in agriculture increases relative to the capital-output ratio in nonagriculture. More formally,

$$\frac{\frac{K^G}{P^G Y^G}}{\frac{K^M}{P^M Y^M}} = \Psi \frac{\alpha}{1-\alpha} \left( \frac{1+\Xi}{\Xi} - \frac{\frac{\alpha}{1-\alpha}\left(\frac{1+\Xi}{\Xi}\right)+1}{\frac{1}{k^M}\chi + \frac{\alpha}{1-\alpha}} \right),$$

which is increasing in $\chi$.

**Stage 3 (Neoclassical).** As the process of capital accumulation proceeds, the labor reserve in traditional agriculture becomes eventually exhausted. This happens when

$$\bar{\chi} = \frac{\beta}{\beta+\Xi} \left( \frac{(1-\zeta)}{\beta\zeta} \right)^{\frac{1}{1-\beta}} \left( 1 + \frac{(1-\alpha)\,\Xi}{\alpha\,(1-\beta)} \right) > \underline{\chi}.$$

For any $\chi > \bar{\chi}$, $\nu^S = 0$ and $\upsilon = 1$. Henceforth, the economy exhibit standard properties of neoclassical models. In particular, if $\varepsilon > 1$ and $\beta > \alpha$, the nonagriculture sector grows in relative size, capital share (i.e., $\kappa$ increases) and expenditure share. The productivity gap remains constant at $\beta/\alpha$ and the relative (agriculture vs. nonagriculture) capital-output ratio is also constant. During this stage, the interest rate falls and the real wage increases as capital accumulates.

**Proposition 4** *Suppose $\varepsilon > 1$, $\beta > \alpha$ and $\omega \to \infty$. Then, as $\chi$ grows, economic development progresses through three stages:*

1. ***Early Lewis:*** *If $\chi \leq \underline{\chi}$, then, $\nu^{AM} = \upsilon = 0$, $\kappa = 1$. Moreover, $\nu^M$ is increasing and $\nu^S$ is decreasing in $\chi$. The interest rate is decreasing and the wage rate is increasing in $\chi$. The (average labor) productivity gap is constant and equal to $1/\alpha$.*

2. ***Advanced Lewis:*** *If $\chi \in [\underline{\chi}, \bar{\chi}]$ then, $\nu^M$ and $\nu^{AM}$ are increasing linearly in $\chi$ while $\nu^S$ is falling linearly in $\chi$ (cf. equation (26)). Therefore, $\upsilon$ is increasing in $\chi$. Moreover, $\kappa$ is decreasing in $\chi$ (cf. equation (25)). The capital-labor ratio in nonagriculture and modern agriculture is constant (cf. equation (24)), but the relative nonagriculture-to-agriculture capital-output ratio is falling in $\chi$. The interest rate and the wage rate are constant, implying that the aggregate labor income share is falling. The (average labor) productivity gap is monotonically decreasing.*

3. ***Neoclassical:*** *If $\chi \geq \bar{\chi}$, then, $\nu^S = 0$ and $\upsilon = 1$. $\nu^M$ is increasing and $\nu^{AM}$ is decreasing in $\chi$. Moreover, $\kappa$ is increasing in $\chi$. The capital-labor ratio is increasing in $\chi$ in both nonagriculture and modern agriculture, but the relative nonagriculture-to-agriculture capital-output ratio is constant. The interest rate is decreasing in $\chi$ and the wage rate is increasing in $\chi$, while the aggregate labor income share is falling. The (average labor) productivity gap is constant. As $\chi$ becomes arbitrarily large, $\kappa \to 1$, $\nu^M \to 1$ and the expenditure share of agriculture tends to zero.*

Figure 6 describes economic transition as capital accumulates. Each panel has $\chi$ on the x-axis. Panel a plots the share of labor in each sector. The labor share in the traditional sector ($\nu^S$) starts high and declines with $\chi$ in the Early and Advanced Lewis stages. The labor share in nonagriculture increases throughout the transition. The labor share in modern agriculture is zero in Early Lewis stage, takes off in the Advanced Lewis stage, and declines again in the neoclassical stage.

Panel b plots factor prices. The interest rate falls and the wage rate increases during the Early Lewis stage. Wages and interest rates are constant during the Advanced Lewis stage, while interest rate falls and wages increase during the Neoclassical stage.

Panel c plots the productivity gap, i.e., the average labor productivity in nonagriculture relative to agriculture. This ratio is constant during the Early Lewis stage, falling in $\chi$ during the Advanced Lewis stage, and constant again in the Neoclassical stage.

Panel d plots the relative capital-output ratio in agriculture to nonagriculture, i.e., $\frac{K^G}{P^G Y^G}\left(\frac{K^M}{P^M Y^M}\right)^{-1}$. The ratio stays at zero during the Early Lewis stage. It increases during the Advanced Lewis stage. Eventually, it becomes constant during the Neoclassical stage.*

## 3.4 Discussion of the Assumptions and of the Mechanics of Economic Growth

In this section, we discuss the plausibility of the different assumptions. The assumptions that $g^M \geq g^{AM} \geq g^S$ and that $\beta > \alpha$ are in line with the common wisdom that manufacturing is the

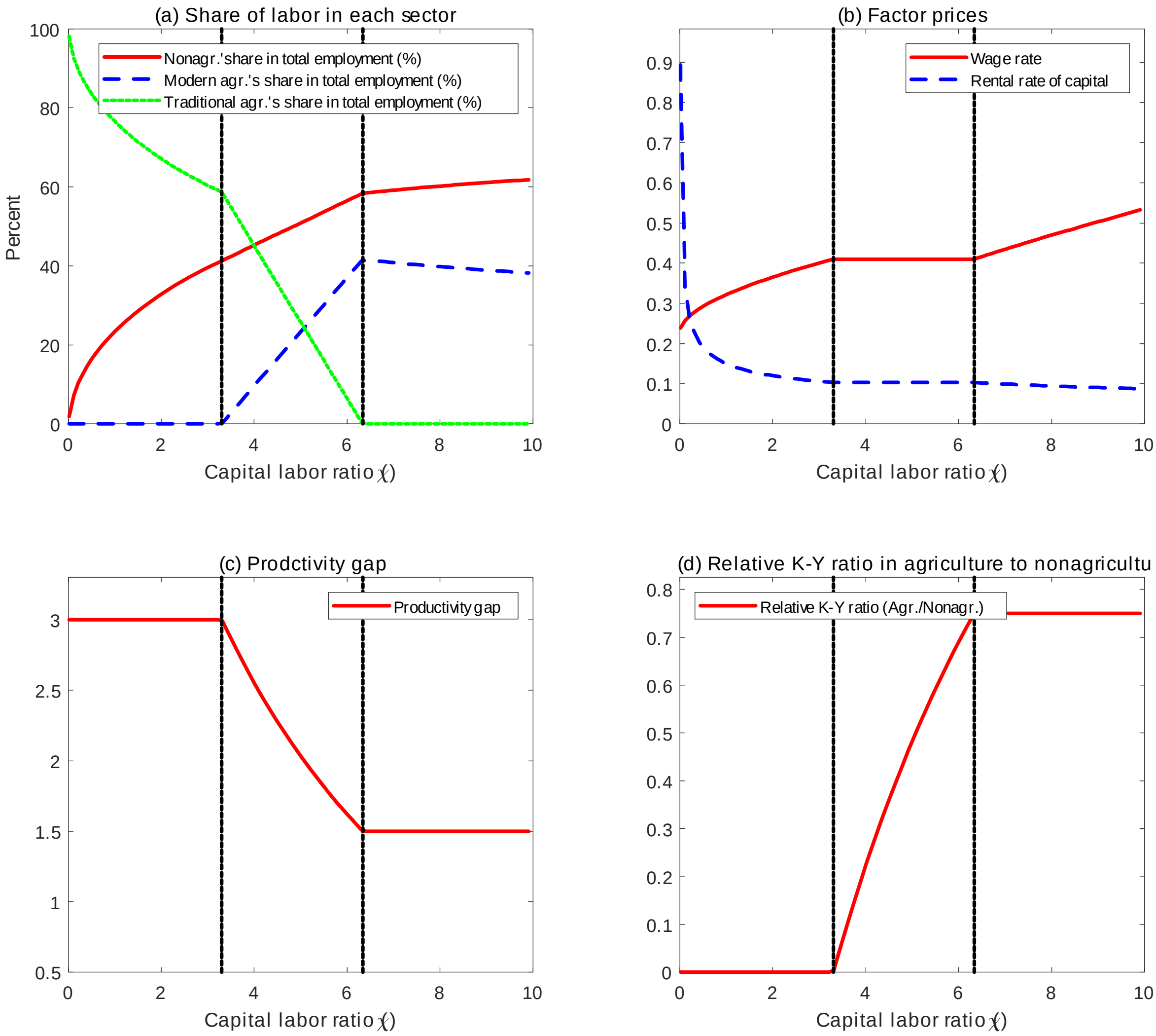



Figure 6: The figure illustrates the allocations and prices as a function of capital per worker, $\chi$, in a version of the Lewis model ($\omega \to \infty$ and no technical change).

most dynamic sector of the economy and that capital-intensive agricultural activities experience more technical progress (e.g., the invention of new machines) than traditional labor-intensive activities. Our estimates confirm this wisdom.

However, the assumption that $\varepsilon > 1$ deserves further scrutiny. Using a value added approach, (Herrendorf, Rogerson, and Valentinyi 2013) estimate a very low elasticity of substitution between manufacturing, services and agriculture in the US. This would suggest a violation of assumption (iii). However, their estimates hinge on a specification in which the elasticity of substitution is assumed to be the same across any two of three sectors. In Section 3.5 below, we show that the data reject the assumption of a low elasticity of substitution in a two-sector model with agriculture and nonagriculture. In particular, the correlation in Figure 1 (Panel d) implies that $\varepsilon > 1$.

Next, we discuss the engines of structural change in our model. The growth of total factor productivity sustains capital accumulation as in standard neoclassical growth models (see e.g., (Cass 1965), (Uzawa 1962), (Ngai and Pissarides 2007)). The relative decline of agriculture follows from the assumption that technical progress is faster in nonagriculture than in agriculture, given an elasticity of substitution larger than unity. The relative price of nonagricultural goods increases over time, but not very fast, so the demand for capital and labor grows faster in nonagriculture and the GDP share of agriculture falls over time. While this effect is well understood, in our model capital accumulation entails an additional novel effect: It induces reallocation within the agriculture sector, away from the labor intensive traditional sector. Such reallocation does not hinge on any assumed difference in the speed of technical progress between the two sectors (i.e., it also occurs if $g^{AM} = g^{S}$). Rather, it stems from an economy-wide capital deepening.

### 3.5 On the Elasticity of Substitution between Agricultural and Nonagricultural Goods

Before estimating the model, we return to the elasticity of substitution between agricultural and nonagricultural goods, denoted by $\varepsilon$ in Equation 1. This is a key parameter, since the results of Proposition 3 and the nature of structural change hinge on $\varepsilon > 1$. To show that this assumption is in line with the evidence, we derive a testable implication from the equilibrium conditions of the model. This is a general condition that does not depend on any specific detail of our theory. It can be derived from a simple production function approach where agricultural and nonagricultural goods enter a CES aggregate production function, as in Equation 1.

Assume the price of the aggregate consumption good $Y$ to be the numeraire. Then, the following standard isoelastic demand condition obtains:

$$\frac{P_t^G}{P_t^M} = \frac{\gamma}{1-\gamma}\left(\frac{Y_t^G}{Y_t^M}\right)^{-\frac{1}{\varepsilon}}.$$

Rearranging terms, and taking log on both sides yields

$$\ln\left(\frac{P_t^G Y_t^G}{P_t^M Y_t^M}\right) = \ln\left(\frac{\gamma}{1-\gamma}\right) + \frac{\varepsilon-1}{\varepsilon}\ln\left(\frac{Y_t^G}{Y_t^M}\right).$$

We take this equilibrium condition to the data. Since, empirically, one cannot reject the hypothesis that the logarithm of the expenditure and output ratio feature unit roots, it is appropriate to take first differences. This yields:

$$\Delta\ln\left(\frac{P_t^G Y_t^G}{P_t^M Y_t^M}\right) = \frac{\varepsilon-1}{\varepsilon}\cdot\Delta\ln\left(\frac{Y_t^G}{Y_t^M}\right). \tag{27}$$

The model predicts a positive correlation between the first differences of the expenditure ratio and that of the real output ratio if $\varepsilon > 1$. The correlation is instead negative if $\varepsilon < 1$. Figure 1 (panel d) and Figure 13 in the appendix provide scatter plots of the five-year changes for the US and China, respectively. In both cases, the correlation is strongly positive, indicating that $\varepsilon > 1$.[16]

(Herrendorf, Rogerson, and Valentinyi 2013) estimate a three-sector CES production function where the elasticity of substitution between agricultural goods, manufacturing goods, and services is by assumption the same. To compare our results with theirs, we extend our model by assuming that nonagriculture comprises two sectors: manufacturing and services. More

[16] Simple OLS regressions on US data in line with equation (27), imply estimates of $\varepsilon$ of 3.3 and 2.0 based on 5-year and annual changes, respectively. For China the corresponding estimates are $\varepsilon = 4.1$ and $\varepsilon = 6.8$ for 5-year and annual, respectively (sample period: 1985-2012). The total changes over the entire 1985-2012 period imply $\varepsilon = 4.2$. In all cases, the data strongly reject the hypothesis that agricultural and nonagricultural goods are complements.

formally, we assume that $Y^M = \left[\hat{\gamma}\left(Y^{Manuf}\right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} + (1-\hat{\gamma})\left(Y^{Serv}\right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}}\right]^{\frac{\epsilon_{ms}}{\epsilon_{ms}-1}}$, where the superscripts $Manuf$ and $Serv$ denote manufacturing and services, respectively, and the parameter $\epsilon_{ms}$ represents the elasticity of substitution between services and manufacturing in production of the nonagricultural good. Then, we estimate our model using the same sectoral value added data (and the same estimation procedure) used by (Herrendorf, Rogerson, and Valentinyi 2013). More formally, we estimate the following production function:

$$Y = \left[\gamma\left(\left[\hat{\gamma}\left(Y^{Manuf}\right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} + (1-\hat{\gamma})\left(Y^{Serv}\right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}}\right]^{\frac{\epsilon_{ms}}{\epsilon_{ms}-1}}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(Y^{G}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

This specification is similar to that used in a different context by (Krusell, Ohanian, Rios-Rull, and Violante 2000). Note that the generalized model nests the specification of (Herrendorf, Rogerson, and Valentinyi 2013) in the particular case in which $\epsilon_{ms} = \varepsilon$. Since their model allows for nonhomothetic preferences with a Stone-Geary demand system, we also generalize preferences in the same direction to make sure that the two models are perfectly nested (see the appendix for details).

We estimate $\epsilon_{ms}$ to be close to zero, indicating a high degree of complementarity between manufacturing and services, consistent with (Herrendorf, Rogerson, and Valentinyi 2013). In contrast, we obtain an estimate of the agriculture-nonagriculture elasticity $\varepsilon = 2.36$, significantly larger than unity. Thus, our model strongly rejects the null hypothesis that $\epsilon_{ms} = \varepsilon$, confirming a high elasticity of substitution between agricultural and nonagricultural goods. Nonhomotheticity is found to be unimportant for the agricultural good, while a significant constant is estimated for the service sector.

Since the focal point of our theory is the decline and modernization of the agricultural sector, in the rest of the paper we ignore the distinction between manufacturing and services.

# 4 Estimating the Model

In this section, we estimate the model. We generalize the model in three dimensions. First, we use discrete time. Second, we introduce an endogenous labor supply choice as in Equation (6).

Third, we introduce land as a fixed factor in modern agriculture.[17] We could also add land in the traditional sector. However, in the spirit of Lewis (1954), we want to retain the property of a traditional sector working as a labor force reserve at a constant marginal cost. Formally, this is important for the model to predict a high elasticity of substitution between capital and labor in agriculture at early stages of development. A formal description of the discrete time model is provided in the technical appendix.

Our strategy comprises the following two steps. In the first, we estimate the deterministic model with constant productivity growth in each sector. To this end, we start by calibrating some parameters externally. Then, we estimate the structural parameters and initial conditions to match moments of the structural change of China between 1985 and 2012. In the second step, we introduce productivity shocks. We estimate stochastic processes for the three TFP shocks and simulate the model. Then, we evaluate the ability of the stochastic model to account for the business cycle properties of China. Finally, we use the estimated model to predict how the properties of this business cycle will evolve as the development process of China progresses further. The results can be compared with the cross-country regularities documented in Section 2.[18]

**Parameters calibrated externally:** On the preference side, we assume a 4% annual time discount rate. Note that the log preferences in Equation (6) ensures that there is no trend in labor supply on the long-run balanced growth path. $\theta$ is chosen so that in the long run agents work one third of their time. We set the annual population growth rate ($n$) to 1.5% following (Acemoglu and Guerrieri 2008). This parameter has no significant impact on the results. Capital is assumed to depreciate at a standard 5% annual rate.

**Estimated parameters based on nonagricultural production data:** We set $\alpha = 0.5$ to match the labor-income share in the nonagricultural sector in China (see (Bai and Qian 2010)). Then, we estimate $g^M$ using standard growth accounting based on a Cobb Douglas production function – consistent with the model – to match the trend of real value-added based GDP in

[17] We assume $Y^{AM} = \left(K^{AM}\right)^{1-\beta-\beta_T} \left(Z^{AM} H^{AM}\right)^{\beta} T^{\beta_T}$, where $T$ is land and $\beta_T$ is the output elasticity of land.

[18] Alternatively, we could have used time-series data for the US to estimate the model. We prefer to use the data for China because the available time-series for the US cover a period when the employment in agriculture is already quite low (20.3% of the total US employment in 1929) and the subsistence activity captured by our traditional agriculture sector is arguably tiny. In contrast, China has a large and declining share of employment in agriculture ranging from 62.4% in 1985 to 33.6% in 2012.

the nonagricultural sector of China between 1985 to 2012. This yields $g^M = 6.5\%$.

**Estimated parameters based on endogenous moments:** We estimate the remaining parameters using the Simulated Method of Moments. The parameters we estimate are

$$\left\{\varepsilon, \omega, \tau, \theta\,\gamma, \varsigma, \beta, \beta_T, g^{AM}, g^S, Z^M_{1985}, Z^{AM}_{1985}, Z^S_{1985}\right\}.$$

We normalize $Y_{1985} = 1$ and target (the natural logarithms of) the following empirical annual observations from 1985 to 2012 in China:[19] (i) the share of agricultural employment in total employment; (ii) the share of capital in agriculture relative to the total capital stock; (iii) the ratio of real output in agriculture to total GDP; (iv) the relative value added share of agriculture, evaluated at current prices (i.e., the expenditure share of agricultural goods); (v) the aggregate GDP growth; (vi) the initial and final aggregate capital-output ratio; and (vii) the 1985-2012 change in the productivity gap between agriculture and nonagriculture, adjusted for rural-urban wage differences. We calculate this gap as the ratio of labor productivity in nonagriculture to agriculture times the ratio of wages in agriculture to nonagriculture. In our model this gap is equal to $(1 - \upsilon(1-\beta))/\alpha$, i.e., the ratio of the labor-income share in agriculture to the labor-income share in nonagriculture.[20].

This yields 143 moment conditions – 28 for each of the annual moments (i)-(v) plus moments (vi)-(vii). The estimation is based on equal weights on the annual moments (i)-(v) and equivalent weights for moments (vi) and (vii) (i.e., 14 times the weight on the annual moments for each of the two K/Y ratios, and 28 times for the relative deviation of the output gap).

We measure real output in agriculture and nonagriculture following the same approach as China's National Bureau of Statistics. The National Bureau of Statistics measures real growth in agricultural and nonagricultural output using prices in a base year. The base year was updated in 1980, 1990, 2000, 2005, and 2010. The levels of real sectoral output are chained when a new

[19] We choose the year 1985 as our initial period because it was a turning point in internal migration policy. In earlier years restrictions on labor mobility between rural and urban areas were very severe. These restrictions were relaxed in January 1985, following the issuance of the "Ten Policies on Further Active Rural Economy" from the CPC Central Committee and the State Council.

[20] We could alternatively have targeted the empirical labor-income shares directly. However, such data are available only up until 2003. From 2004 onwards, the labor income shares are not comparable to their counterparts in the pre-2004 period. See (Bai and Qian 2010) for details. As it turns out, the overall change in the ratio of labor income shares is comparable to the change in the labor productivity gap: over the 1985-2003 period these ratios fall by 13 and 14 log points, respectively.

base year is adopted. Therefore, in the year of change in base year prices the real levels are by construction invariant to using the new or the old set of prices. This approach is similar to that pursued in the U.S. National Income and Product Accounts before 1996.[21]

**Nonhomothetic preferences:** We also estimate the model under an important generalization of the theory. Our benchmark theory postulates homothetic preferences. However, earlier studies point at a potentially important role for income effects (see, e.g., (Boppart 2014)). Therefore, we estimate a version of the model featuring generalized Stone-Geary preferences with a minimum per-capita consumption of the agricultural good, as in (Herrendorf, Rogerson, and Valentinyi 2013). To this end, we reinterpret the goods $M$ and $G$ as final goods that are allocated to consumption (denoted $C^M$ and $C^G$) and investment (denoted $X^M$ and $X^G$), where $Y^G = C^G + X^G = Nc^G + X^G$ and $Y^M = C^M + X^M = Nc^M + X^M$. More formally, we replace the per-capita utility function in equation (6) by

$$u\left(c^G, c^M, h\right) = \theta \log\left(\left[\gamma\left(c^G - \bar{c}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(c^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right) + (1-\theta)\log(1-h),$$

while the investment good continues to be a CES aggregation of the agricultural and nonagricultural goods,

$$\left[\gamma\left(X^G\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(X^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} = I.$$

The aggregate resource constraint (7) can then be written as

$$\dot{K} = I - \delta K - \tau \bar{W} H^M + Tr. \tag{28}$$

Finally, we denote the total expenditure by $Y = P^G Y^G + P^M Y^M$, the total consumption expenditure as $C = P^G C^G + P^M C^M$, and the total investment expenditure as $I = P^G X^G + P^M X^M$. Note that we obtain the baseline model as a special case when $\bar{c} = 0$.

Table 1 summarizes the results of the estimation.

The estimated elasticities of substitution $\varepsilon$ and $\omega$ are in all cases significantly larger than unity, consistent with the assumption of Proposition 3 (see also Section 3.5 above). In the

[21] To be consistent with the empirical data, we start the model in 1980 and base the 1985-1990 growth rates using 1980 as the base year, then update and chain the output levels using 1990 relative prices, etc.

| | Parameters Set Exogenously | Homothetic | Nonhomothetic |
|---|---|---|---|
| $n$ | growth rate of 15+ population | 1.5% | 1.5% |
| $\delta$ | capital depreciation rate | 5% | 5% |
| $(1+\rho)^{-1}$ | discount factor | 0.96 | 0.96 |
| $\theta$ | preference weight on consumption | 0.73 | 0.73 |
| | **Parameters Estimated Within the Model** | | |
| | Targeting Data on nonagricultural Production | Homothetic | Nonhomothetic |
| $\alpha$ | labor share in nonagriculture | 0.50 | 0.50 |
| $g_M$ | nonagriculture TFP growth rate | 6.5% | 6.5% |
| $Z^M_{1985}$ | initial TFP level in nonagriculture | 4.33 | 4.45 |
| | Targeting Empirical Moments on Structural Change | Homothetic | Nonhomothetic |
| $\bar{c}$ | Subsistence level in food consumption | – | 0.05 |
| $\varepsilon$ | ES btw agric. and nonagric. consumption | 3.60 | 3.36 |
| $\omega$ | ES btw modern-agric. and traditional-sector output | 9.00 | 9.00 |
| $\gamma$ | weight on agric. intermediate goods | 0.61 | 0.60 |
| $\varsigma$ | weight on modern-agriculture output | 0.40 | 0.39 |
| $1-\beta-\beta^{LAND}$ | capital's income share in modern-agric. | 0.14 | 0.13 |
| $\beta$ | labor's income share in modern-agric. | 0.61 | 0.60 |
| $\tau$ | labor wedge between agric. and nonagriculture | 0.76 | 0.75 |
| $g^{AM}$ | TFP growth rate in modern-agric. 1985-2012 | 6.1% | 6.1% |
| $g^S$ | TFP growth rate in traditional sector 1985-2012 | 0.9% | 0.9% |
| $Z^S_{1985}$ | initial TFP level in traditional agric | 1.23 | 1.18 |
| $Z^{AM}_{1985}$ | initial TFP level in modern-agric. | 2.26 | 2.25 |

Table 1: Calibration Results

homothetic economy $\varepsilon$ is slightly lower than that implied by estimating equation (27) on Chinese data (where we obtain $\varepsilon \geq 4$ with the exact estimate depending on the data frequency).

When we allow nonhomothetic preferences, the estimated $\varepsilon$ falls slightly. The reason why the nonhomothetic model features a lower estimate for $\varepsilon$ is that with homothetic preferences the only mechanism for the model to generate a falling expenditure share on agricultural goods as the relative output share of agriculture shrinks is the fall in the relative prices of nonagricultural goods. To match the data, the price elasticity of the relative demand must be high in this case. In contrast, in the presence of a subsistence level in agricultural goods, the expenditure share in agriculture also falls because of an income effect. Therefore, the model is consistent with a lower price elasticity of the relative demand, or a lower estimated $\varepsilon$. However, the estimated subsistence level $\bar{c}$ turns out to be very small, corresponding to 5% of GDP in 1985. Therefore, the estimated $\varepsilon$ falls only slightly, and the remaining parameters are also very similar to the homothetic case.

We also note that the elasticity of substitution between modern and traditional agriculture, $\omega$, is very large. The productivity growth rate is very high in both manufacturing and modern agriculture. This reflects the high growth rate of the Chinese economy that our model targets. To avoid the (to us) unrealistic implication of a long run growth rate exceeding 6%, we assume the productivity growth rate in both nonagriculture and modern agriculture to gradually (linearly) decline to 1.8% over the 2012-2112 period. Thereafter, the economy grows at an annual 1.8% growth rate. This assumption has negligible effects on the quantitative results. Finally, we note that the estimated productivity growth in modern and traditional agriculture satisfy the (sufficient) conditions for convergence to the ABGP set forth in Proposition 3.

## 4.1 Accounting for Structural Change In China

In this section, we show that the benchmark model fits well the data along the salient dimensions of the process of structural change. Figures 7 and 8 plot the time series for the seven empirical targets against the implications of the estimated model (with and without homothetic preferences). The model fits well the process of structural change of China: it predicts that the shares of employment, capital, value added, and expenditure in the agricultural sector relative to total should be falling over time, see Figure 7.

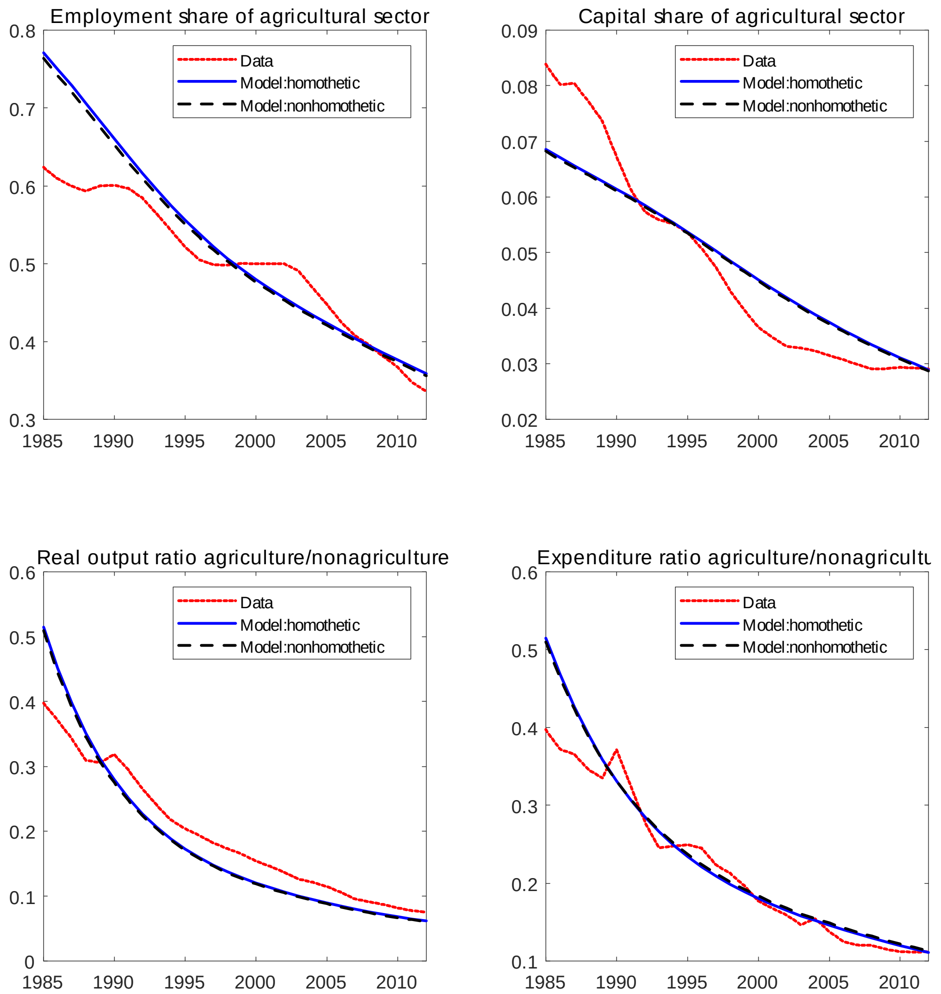



Figure 7: Structural change in model versus targeted empirical moments. The homothetic model is solid lines while the nonhomothetic model is dashed lines. The top left panel displays agricultural employment as a share of total employment. The top right panel displays the share of aggregate capital invested in agriculture. The bottom left panel displays the agricultural value added as a share of aggregate GDP at current prices. The bottom right displays the expenditure on agricultural goods as a share of aggregate GDP.

Figure 8 shows two aggregate variables, an index of the log GDP per capita (index) and the capital-output ratio, and the productivity gap measured by the output per worker in agriculture relative to total output per worker. It shows in addition the demeaned log of the productivity gap between nonagriculture and agriculture. The estimated model captures well the trend in GDP and capital-output ratio. It also captures the falling productivity gap, although in the data there are large swings, most notably an increase during the period 1995-2002.

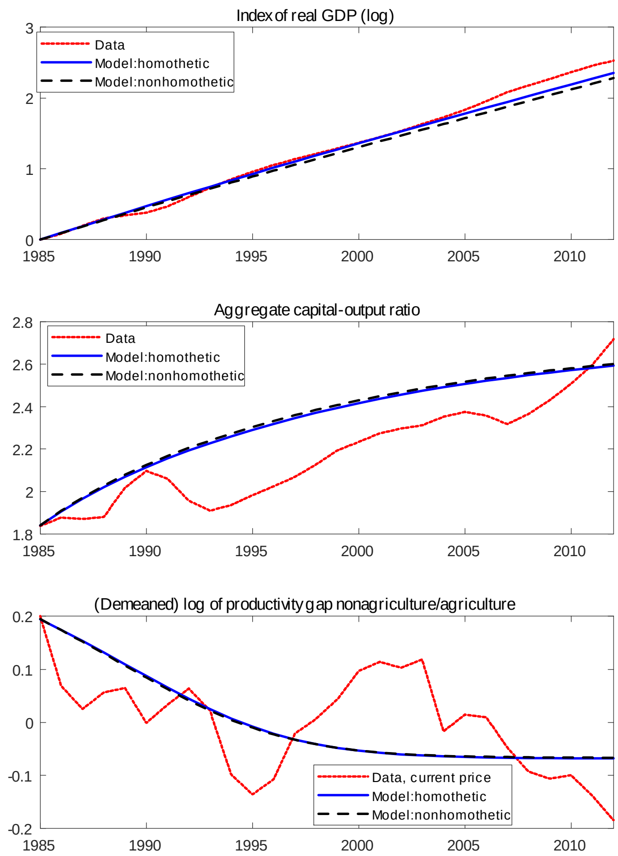



Figure 8: Structural change in model versus targeted empirical moments. Solid blue lines: homothetic model. Dashed blue lines: nonhomothetic model is dahsed lines. The top panel displays an index of real GDP (logarithm). The middle panel displays the capital-output ratio. The bottom panel displays changes the labor productivity gap.

Finally, Figure 9 illustrates the prediction of the model about the demise of the traditional agriculture. The employment share of the modern sector in total agriculture increases from about 25% in 1985 to almost one in 2012. The output share (corresponding to the variable $\upsilon$

in the model) exhibits a similar behavior. Note that we do not have direct data on traditional versus modern agricultural sectors, so the transition from traditional to modern agriculture is identified from the change in the productivity gap (the bottom panel of Figure 8). In the model, the decline of the traditional sector is due to both fast TFP growth (that is exogenous in our model) and fast capital accumulation (that is endogenous in our model). In particular, capital accumulation triggers a shift away of the traditional sector since it is labor intensive.

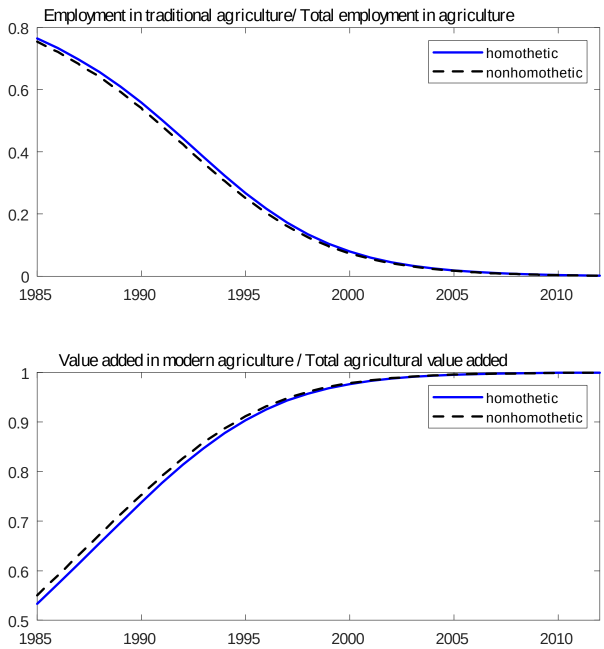



Figure 9: Structural change, according to the models. Dashed blue lines: homothetic model. Dotted black lines: nonhomothetic model. Panel A displays the share of agricultural employment working in traditional sector. Panel B displays the evolution of $\upsilon$; value added in modern agriculture as a share of total agricultural value added, in current prices.

## 4.2 Taking Stock

...

# 5 Business Cycle Analysis

In this section, we introduce uncertainty in the form of shocks to the three TFPs. In the robustness analysis we also study the case with only two shocks.

## 5.1 Estimating the stochastic process for TFP

The estimation of the process for technology shocks presents a measurement problem because we do not have direct observations of traditional versus modern agriculture. To overcome this problem we impose two natural equilibrium conditions, namely that the marginal product of capital is equated across manufacturing and modern agriculture, and that the marginal product of labor is the same in the two agricultural sectors. Combining these conditions with direct annual observations of capital, labor, and value added in agriculture and non-agriculture, we can uniquely identify the sequence $\left\{Z_t^M, Z_t^{AM}, Z_t^S\right\}_{t=1985}^{2012}$.

We decompose the TFP level into a trend and a cyclical part, assuming that the trend is deterministic. to this aim, define $z_t^j \equiv \log\left(Z_t^j\right) - \log\left(\bar{Z}_t^j\right)$, where $\bar{Z}_t^j = \bar{Z}_0^j\left(1+g^j\right)^t$ is the deterministic trend for $j \in \{M, AM, S\}$. We assume that the stochastic process is VAR(1), such that

$$\begin{bmatrix} z_{t+1}^M \\ z_{t+1}^{AM} \\ z_{t+1}^S \end{bmatrix} = \begin{bmatrix} \phi^M & 0 & 0 \\ 0 & \phi^{AM} & 0 \\ 0 & 0 & \phi^S \end{bmatrix} \cdot \begin{bmatrix} z_t^M \\ z_t^{AM} \\ z_t^S \end{bmatrix} + \epsilon_t, \tag{29}$$

where $\epsilon_t = A \cdot \tilde{\epsilon}_t$, $A$ is a $3 \times 3$ matrix and $\tilde{\epsilon}_t$ denotes a vector of orthogonal i.i.d. shocks. Off-diagonal elements in the matrix $A$ will capture correlation between the three TFP innovations $\epsilon_t^j$.

The estimated parameters are: $\hat{\phi}^M = 0.63$, $\hat{\phi}^{AM} = 0.9$, and $\hat{\phi}^S = 0.42$, all significant at 1% level.[22] The persistence of the three shocks on a quarterly basis, is 0.89, 0.97, and 0.81,

[22] Note that the autocorrelation matrix has no off-diagonal elements. If we allow for off-diagonal elements in this matrix, we obtain:

$$\begin{aligned} \log z_{t+1}^M &= 0.72^{***} \log z_t^M - 0.07 \log z_t^A + 0.12 \log z_t^S + \varepsilon_{t+1}^M \\ \log z_{t+1}^A &= -0.03 \log z_t^M + 0.906^{***} \log z_t^A - 0.05 \log z_t^S + \varepsilon_{t+1}^A \\ \log z_{t+1}^S &= 0.10 \log z_t^M + 0.02 \log z_t^A + 0.438^{***} \log z_t^S + \varepsilon_{t+1}^S \end{aligned}$$

Since all off-diagonal coefficients are insignificant, we set them to zero and estimate a restricted VAR.

respectively. The estimated covariance matrix of $A$ correlation matrix is

$$Corr\left(A\right)=\begin{bmatrix}1 & 0.42^{**} & -0.48^{**}\\ 0.42^{**} & 1 & -0.15\\ -0.48^{**} & -0.15 & 1\end{bmatrix}.$$

This implies that innovations to nonagriculture and modern agriculture are positively correlated, whereas innovations to nonagriculture and traditional agriculture are negatively correlated. Finally, the implied standard deviation of the innovations in (29) are given by $\sigma\left(\epsilon_t^M\right)=0.042$, $\sigma\left(\epsilon_t^{AM}\right)=0.036$, and $\sigma\left(\epsilon_t^S\right)=0.053$.

We assume that the realization of the stochastic productivity shock is observe after capital is installed in each sector. Therefore, capital can only adjust in the following period.

## 5.2 Simulating the stochastic economy

We simulate the model using the estimates in Table 1 that were obtained to match the structural change of China, assuming that the economy starts with the initial condition of 1985. We augment the model with the stochastic process for TFPs discussed above.

Solving the model is a nontrivial task. We cannot approximate the economy around a balanced growth path. Instead, we proceed as follows. We first solve for a stochastic one-sector version of our model without agriculture, using standard methods. We assume that our benchmark three-sector model converges to this one-sector model after 250 periods. Proposition 3 shows that the ABGP of our benchmark economy indeed converges to the balanced growth path of this one-sector model. We then solve the economy recursively for each time period, back to period $t=0$.

The stochastic process for $z_t\equiv[z_t^M,z_t^{AM},z_t^S]'$ is approximated by a 27-state Markov chain with three realizations for each shock, using a standard Tauchen method (Tauchen 1986). There are two continuous state variables, $\kappa$ and $\chi$. We approximate the next-period decision rules for $(c_{t+1},h_{t+1})$ with piecewise linear functions over the state variable $\left(\kappa_{t+1},\chi_{t+1},z_{t+1}\right)$. We solve for the optimal decisions on a grid with 75 grid points for $\kappa$ and $\chi$. The location of this grid is adjusted over time. In period $t$ the grid for $\chi_t$ is distributed from $0.90\bar{\chi}_t$ to $1.1\bar{\chi}_t$ where $\bar{\chi}_t$

denotes the deterministic trend. Similarly, the grid for $\kappa_t$ distributed from $\bar{\kappa}_t - 0.025$ to $\bar{\kappa}_t + 0.025$. We verify that realized optimal $(\kappa_t, \chi_t)$ never exceeds these brackets. Given decision rules for $(c_{t+1}, h_{t+1})$, the optimal control variables follow from the state and the optimality conditions. In particular, we solve for current-period optimal choices for $\left(\chi_{t+1}, \kappa_{t+1}, h_t, c_t, \nu_t^M, \nu_t^{AM}, \nu_t^S\right)$. The decision rules for $\chi_{t+1}$, $\kappa_{t+1}$, $h_t$, $c_t$, $\nu_t^M$, are approximated by piecewise linear functions over $(\kappa_t, \chi_t, z_t)$ and the decision rules for $\nu_t^{AM}$ and $\nu_t^S$ follow directly from the optimality conditions once the values for $\kappa_t$ and $\nu_t^M$ are determined.

We start each sample economy off with an initial value for $\hat{\chi}_{1980}$ such that the deterministic model reaches the empirical value for $\hat{\chi}_{1985}$ in 1985. Moreover, $\kappa_{1985}$ is set to the 1985 value on the ABGP. We then simulate 1000 versions of the economy starting in 1980 and calculate statistics from 1985 to 2185.

Both the empirical data and the simulated data are filtered to take out the trend. We report results based on HP-filtered data (Table 2 in the main text) and data in First Differences (Tables 7.2 in the appendix). The upper panel of each table presents the business cycle statistics for China 1985-2012. The lower panel reports the same statistics for the simulated economies. Tables 2 reports results based on HP-filter (with an HP parameter of 6.25), whereas Tables 7.2 reports results based on first differences. In the discussion we focus on the HP-filtered data. As is clear from the tables, the empirical and theoretical business-cycle properties are almost invariant to the choice of filter. Figure 10 shows the impulse response functions for employment, value added, and the productivity gap to sectoral TFP shocks. Although the model has correlated shocks, here we illustrate the dynamics to shocks individually, holding the other TFP shocks at the zero level.

**Productivity gap.** The model predicts correlations between the productivity gap and sectoral labor supply that are qualitatively in line with the empirical observation. The productivity gap is overall countercyclical; it decreases when the employment in nonagriculture is high and decreases when the employment in modern agriculture is high. However, the volatility of the fluctuations in the productivity gap is only about one third of its empirical counterpart. Intuitively, a positive productivity shock in nonagriculture attracts workers from agriculture and inducing modernization in the agricultural sector. Interestingly, a cyclical boom works like a temporary acceleration of the process of structural change discussed in the previous section (cf. Lemma 2).

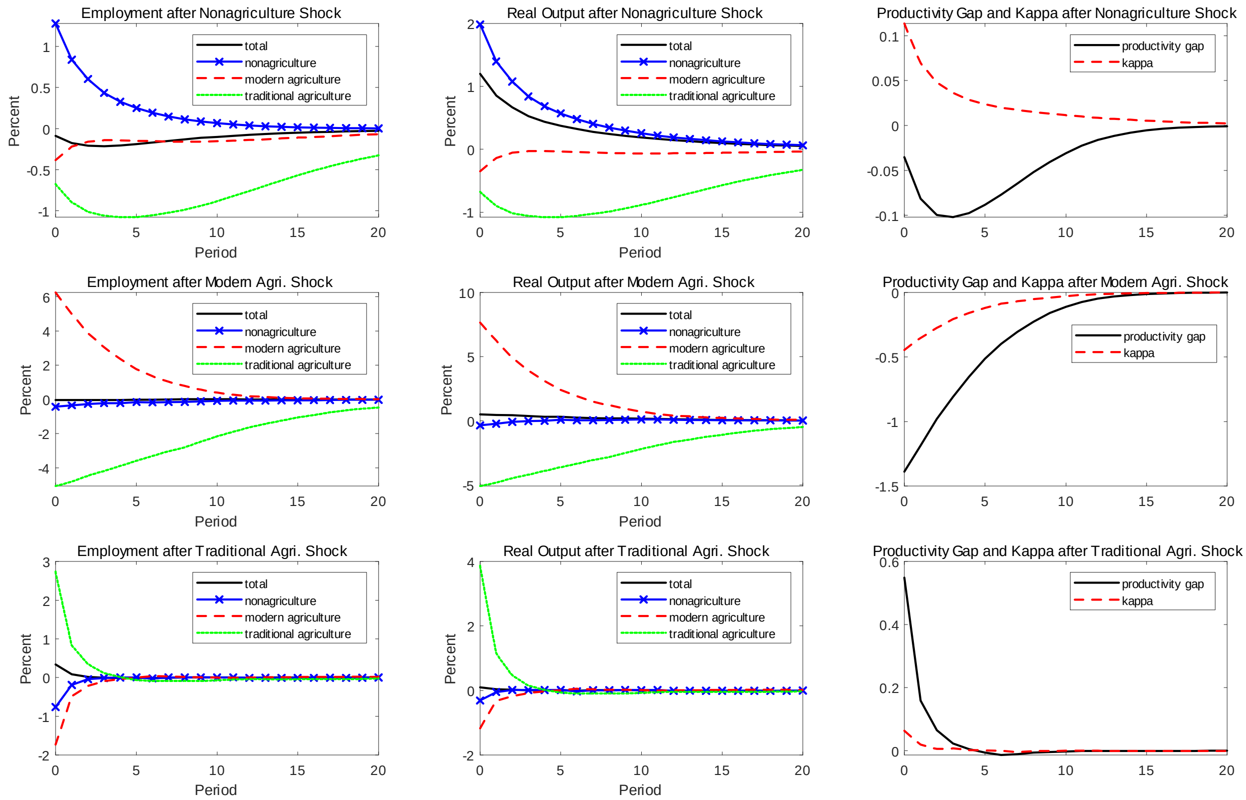



Figure 10: **Impulse Response Functions.** All graphs show impulse response as percentage deviation from the deterministic path. The three top panels show responses to a one-standard deviation change in nonagricultural TFP ($Z^M$). The three middle panels show responses to modern agricultural TFP ($Z^{AM}$). The three bottom panels show responses to traditional agricultural TFP ($Z^S$).

The impulse response functions in Figure 10 illustrate the mechanism. An increase in $Z^M$ (upper panels) triggers a shift of labor (and capital, not shown in the figure) to nonagriculture. In the period when the shock occurs, only labor adjusts. Subsequently, capital accumulates in the more productive sector, sourced from both modern agriculture and net capital accumulation. Labor is sourced from traditional agriculture and, to a lesser extent, from modern agriculture. Thus, labor supply in agriculture falls simultaneously with modernization of agriculture: both the average capital intensity and the average labor productivity increase in agriculture. This in turn causes the productivity gap to fall (upper right panel of Figure 10). The impulse response of sectoral employment shows that a shock to $Z^S$ (bottom panels) has a similar (if opposite) effect on the employment and output dynamics: a higher $Z^S$ causes labor to be reallocated to traditional agriculture sourced from modern agriculture and nonagriculture. This in turn increases the relative size of traditional agriculture in total agriculture thereby lowering the average labor productivity in agriculture and increasing the labor income share. Thus, shocks to $Z^M$ and $Z^S$ induce a positive comovement in the productivity gap and agricultural employment. In contrast, a positive shock to modern agriculture induces an increase in total employment in agriculture and a decline of employment in the traditional sector and in the productivity gap. Thus, shocks to modern agriculture mitigate the countercyclicality of the productivity gap.

**Sectoral labor supply and GDP**. The dynamics of the productivity gap is instructive for understanding the comovement between sectoral labor supply and GDP. Both in the model and in the data (see Table 2) employment in nonagriculture $\nu^M$ is positively correlated with GDP, consumption, and investment whereas employment in agriculture $\nu^G$ is negatively correlated with GDP, consumption, and investment. The reason for the asymmetry between sectors, reflected in opposite signs of the comovements of GDP with $\nu^M$ and $\nu^G$, lies in the presence of misallocation and in the role of the traditional agriculture as a labor reserve. Recall that since $\tau = 0.76$, employment in agriculture is suboptimally too large. The presence of a wedge between urban and rural sectors in China is in line with a number existing studies, see e.g., (Brandt and Zhu 2010), (Cheremukhin, Golosov, Guriev, and Tsyvinski 2017) Thus, reallocation to nonagriculture has a positive efficiency effect. When TFP in nonagriculture increases, $\nu^M$ increases, GDP goes up both by the direct effect of a productivity increase and because of reduction in misallocation. Both capital and labor move towards nonagriculture, with a fall in the capital labor ratio. While the net effect on employment in modern agriculture is ambiguous, the share of agricultural workers employed in modern agriculture unambiguously increases (see Lemma 2).

The labor reserve amplifies reallocation since many workers can leave agriculture without much effect on the marginal product of labor. This effect is reminiscent of the mechanism in Lewis (1954) at business cycle frequencies. Both the reduction in misallocation and the low opportunity cost of sourcing workers from agriculture result in a strong correlation between employment in nonagriculture and GDP. In contrast, when TFP increases in agriculture the positive direct effect of a higher average TFP on GDP is dampened (and possibly offset) by the increasing misallocation. Thus, employment in agriculture and GDP can move in opposite directions. Although the sign of the net effect hinges on parameters, in our calibration the correlation happens to be negative as in the data.

Expenditure and value added exhibit a cyclical pattern similar to that of sectoral labor: expenditure on nonagricultural good is strongly procyclical, while expenditure on agricultural good is acyclical in the data and only weakly procyclical in the model. The single business-cycle aspect that the model gets qualitatively wrong is that it is inconsistent with the empirical observation that agricultural value added is positively (negatively) correlated with nonagricultural labor (agricultural labor) in China.

**Volatility of labor supply and GDP**. The model predicts a low volatility of employment and a low correlation with GDP, although both are larger than in the data (note that employment volatility is particularly low in China even compared with countries at similar stage of development, see Figure 4). Moreover, the model is consistent with the observation that aggregate labor supply is highly correlated with employment in agriculture and approximately uncorrelated with employment in nonagriculture. The low volatility and low correlation of employment with total GDP stem from the fact that labor can be sourced from a large agricultural sector without generating, as discussed above, large fluctuations in wages. Therefore, labor supply is less volatile (and less correlated with GDP) than in a one-sector economy.

**Consumption and investments**. Finally, the model shares many of the features of a standard one-sector RBC model, including that investment is more volatile than output and that consumption volatility is too low in the model. Moreover, with TFP shocks as the only source of uncertainty the model generates the same volatility of GDP as in the data.

| | $c$ | $i$ | $\frac{P^G y^G}{P}$ | $\frac{P^M y^M}{P}$ | $\frac{APL^G}{APL^M}$ | $n^G$ | $n^M$ | $n$ |
|---|---|---|---|---|---|---|---|---|
| | A. HP Filtered China Data, 1985-2012 | | | | | $std\,(y) = 1.7\%$ | | |
| $\frac{std(x)}{std(y)}$ | 0.99 | 3.53 | 1.63 | 1.34 | 2.04 | 0.64 | 0.73 | 0.10 |
| $corr(x, y)$ | 0.70 | 0.65 | 0.06 | 0.95 | -0.17 | -0.69 | 0.73 | -0.23 |
| $corr(x, n^G)$ | -0.60 | -0.31 | -0.37 | -0.55 | 0.48 | 1.00 | -0.94 | 0.48 |
| $corr\left(x, n^M\right)$ | 0.60 | 0.37 | 0.41 | 0.57 | -0.54 | -0.94 | 1.00 | 0.04 |
| | B. HP Filtered Model, Homothetic model | | | | | $std\,(y) = 1.7\%$ | | |
| $\frac{std(x)}{std(y)}$ | 0.27 | 2.39 | 1.09 | 1.18 | 0.62 | 1.03 | 1.07 | 0.42 |
| $corr\,(x, y)$ | 0.81 | 0.99 | 0.30 | 0.97 | -0.38 | -0.25 | 0.73 | 0.43 |
| $corr\left(x, n^G\right)$ | -0.08 | -0.25 | 0.78 | -0.43 | 0.73 | 1 | -0.75 | 0.69 |
| $corr\left(x, n^M\right)$ | 0.45 | 0.75 | -0.31 | 0.87 | -0.74 | -0.75 | 1 | -0.21 |

Table 2: Summary Statistics for China data and Model: HP-filtered

## 5.3 Evolution of the Business Cycle during Structural Change

In this section we simulate the model beyond the stage of structural change reached by China in 2012. This allows to forecast the evolution of business cycles in China and to compare the prediction of the model for an economy at the stage of development of China with that of a fully industrialized economy. We focus on four statistics: (i) the correlation between agricultural employment and productivity gap; (ii) the correlation between agricultural and nonagricultural employment, (iii) the correlation between total employment and GDP, and (iv) the volatility of employment relative to GDP.

The results are shown in Figure 11. Each dot in the figure represents a statistic covering a 28-year rolling window. The first dot on the right of each figure (i.e., that corresponding of the largest agricultural share) calculate statistics for simulated economies corresponding to China in the period 1985-2012, the second dot corresponds to the period the period 1986-2013, and so on. The process of economic development corresponds in each figure to a movement from left to right. A fully industrialized economy is an economy with an employment share of agriculture lower than 10% (recall that in our model the employment share of agriculture goes asymptotically to zero, which is arguably an extreme prediction of the theory).

The top left panel shows that the correlation between agricultural employment and the productivity gap decreases as structural change proceeds (i.e., as the share of agriculture falls). The reason is that as the agricultural sector modernizes and the Lewis sector shrinks, the economy converges to a Hansen-Prescott economy with a constant productivity gap. The productivity

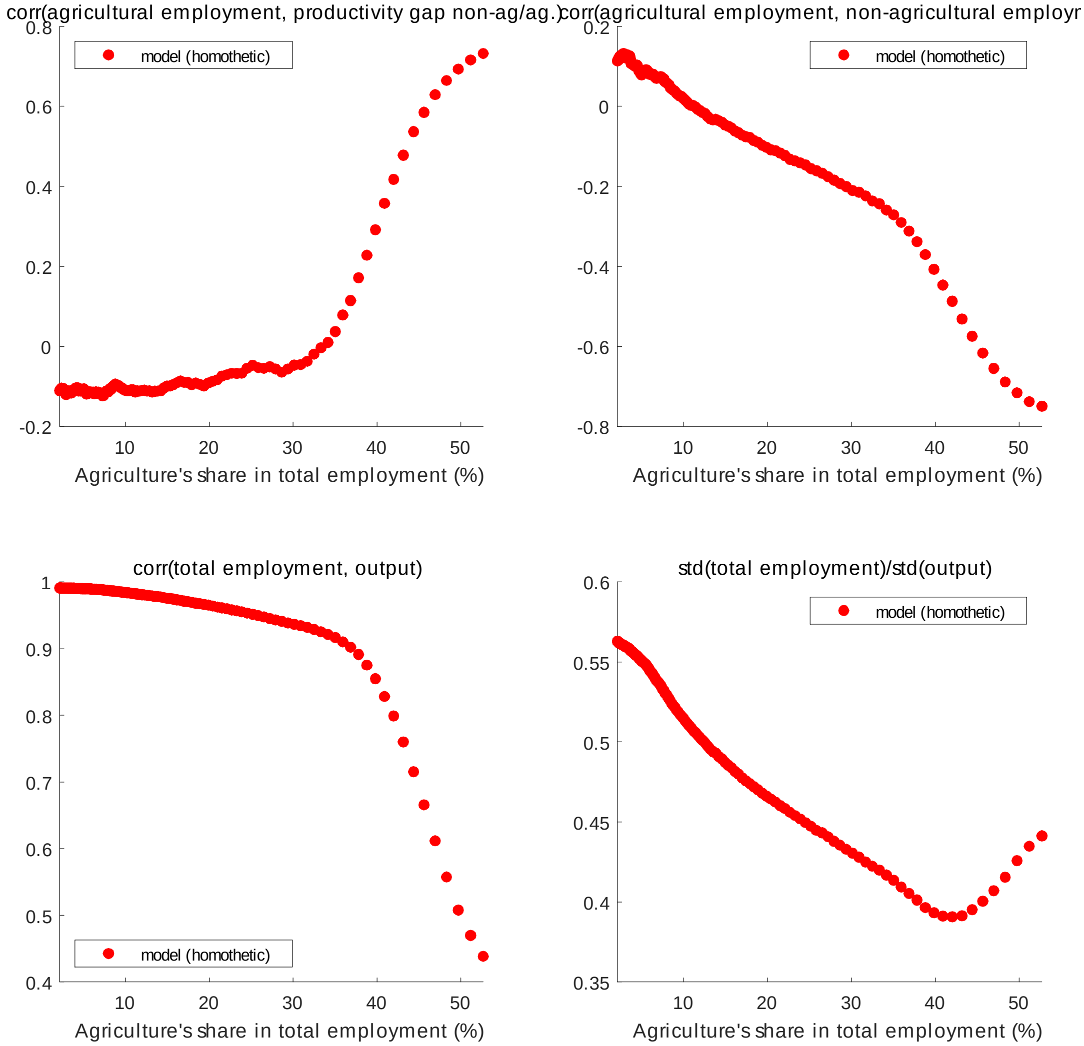



Figure 11: **Business Cycle during Structural Change.** The graphs show the evolution of business cycle statistics as a function of the employment share in agriculture. Each dot shows a statistic covering a 28-year rolling window. Simulated data are HP-filtered. The upper left panel shows the correlation between employment in agriculture and the productivity gap. The upper right panel shows the correlation between employment in agriculture and employment in nonagriculture. The lower left panel shows the correlation between total employment and GDP. The lower right panel shows the volatility of aggregate employment relative to GDP.

gap is then entirely explained by technology parameters and by the exogenous wedge $\tau$.

The top right panel shows that the correlation between employment in agriculture and nonagriculture increases as the share of agriculture falls. In particular, the correlation is negative and large in absolute value as long as the employment share in agriculture is between 40%-50% (which is close to the value for today's China), and is about zero for fully modernized economies. The reason is twofold. First, the presence of a large agricultural sector works as a buffer: when there are sector-specific shocks, labor will naturally move in and out of agriculture. These fluctuations become less pronounced as the agricultural sector shrinks. Second, the Lewis sector provides a labor reserve with a high elasticity of substitution between sectoral employments. As the Lewis sector shrinks, the effective elasticity of substitution across sectors falls.

The bottom left panel shows that the correlation between total employment and GDP is increasing as the share of agriculture falls. The correlation increases from 40% to 100%.[23] The reason is manifold. First, in a multisector economy productivity shocks lead to reallocation of labor across sectors without requiring large swings in the marginal product of labor and wages. In contrast, in the one sector economy wages fluctuate more since the only margin of adjustment is labor-leisure. Therefore, labor's movement across sectors dampens fluctuations in labor supply. Second, an economy with a large agricultural production has a low aggregate capital-output ratio. Therefore, movements in aggregate capital cause large swings in the marginal product of capital making consumption more positively correlated with GDP through a standard Euler equation mechanism. In turn, this lowers the fluctuations in labor supply originating from income effects. Finally, in our calibration of initial conditions the aggregate labor supply declines over time causing an increase in the average Frisch elasticity of labor supply.

The bottom right panel shows that the volatility of employment relative to GDP is lower for an economy like China than in a fully industrialized economy. This is a natural implication of the discussion above of the employment-GDP correlation. However, in our calibration, employment volatility decreases with development at earlier stages of the process of structural change (i.e., when agriculture employs more than 45% of the total hours worked). The reason for this nonmonotone behavior is that when traditional agriculture is large, fluctuations are largely driven by TFP shocks to this sector ($Z^S$). The estimated persistence of the TFP shock in

[23] Recall that in a one-sector business cycle model temporary TFP shocks to nonagriculture are the only source of fluctuations in both employment and GDP. Therefore, the correlation must be unity.

traditional agriculture happen to be low which implies a large response in labor supply. We believe that this result may partly arise from measurement error, which is particularly important in the traditional sector (where, recall, we retrieve TFP shocks indirectly rather than through standard growth accounting, because of lack of data). Measurement error is likely to exaggerate the volatility and underestimate the persistence of shocks to $Z^S$. We return to this in the robustness section.

## 5.4 Robustness analysis

This section explores three robustness analysis exercises: (1) introducing capital adjustment costs; (2) assuming shocks to $Z^S$ have the same persistence of as shocks to $Z^{AM}$; and (3) assuming a large food subsistence level and a unit elasticity between agriculture and nonagriculture ($\varepsilon \approx 1$).

### 5.4.1 Capital adjustment costs

In our benchmark model capital in each sector is set one period in advance, and after one period reallocation of capital between sectors can occur without cost. It is important to investigate the effect of introducing additional costs of reallocation of capital between agriculture and nonagriculture. Indeed, capital adjustment costs are standard in the quantitative DSGE literature (cf. (Christiano, Eichenbaum, and Evans 2005); (Smets and Wouters 2007)), including papers studying business cycles in models with multiple sectors (see e.g. (Horvath 2000b); (Bouakez, Cardia, and Ruge-Murcia 2009) ; (Iacoviello and Neri 2010)).

Following (Bouakez, Cardia, and Ruge-Murcia 2009) and (Iacoviello and Neri 2010) we consider the canonical capital adjustment model where it is costly to change the rate of investment. Recall that for each capital stock $j$, the law of motion for capital is given by $K_{t+1}^j = (1-\delta) K_t^j + I_t^j$, where $j \in \{M, G\}$ and $I_t^j$ is the effective investment in sector $j$. The cost of investment is reflected in an aggregate resource constraint for investment goods,

$$\left[\gamma \left(X_t^G\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) \left(X_t^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} = \Psi_t^G \left(\frac{I_t^G}{K_t^G}, K_t^G\right) + \Psi_t^M \left(\frac{I_t^M}{K_t^M}, K_t^M\right) + I_t^G + I_t^M,$$

where the terms $\Psi_t^{AG}$ and $\Psi_t^M$ reflect the adjustment cost. Recall also that $X_t^i$ is the quantity of good $i \in \{M, G\}$ allocated to investment. We assume that the adjustment cost function $\Psi$ has a standard quadratic form,

$$\Psi_t^j \left( \frac{I_t^j}{K_t^j}, K_t^j \right) = \frac{\xi}{2} \left( \frac{I_t^j}{K_t^j} - \delta - g_t^j \right)^2 K_t^j,$$

where the parameter $g_t^j$ is the growth rate of capital $K_t^j$ in period $t$ in the deterministic structural transition and $\xi$ is a nonnegative adjustment cost parameter. It follows that as long as the capital stock $K_t^j$ grows at exactly the same rate as in the deterministic transition, $g_t^j$, the adjustment costs are zero. However, when the investment rate deviates from this level then quadratic costs are incurred.

We set the adjustment cost parameter to $\xi = 2.5$, which is slightly lower than the annual equivalent to the value of $\xi$ estimated by (Iacoviello and Neri 2010) based on quarterly data for the US ($\xi = 11$). Note that in an annual model it is impossible to change the capital stock more often than annually, so our benchmark economy effectively embeds some investment sluggishness since sectoral capital is set one period in advance. Moreover, note that by assumption the structural change dynamics would not be affected by the capital adjustment cost. We therefore keep all other parameters the same as in the benchmark homothetic economy.

The effects of introducing adjustment costs is that investments become more sluggish. Therefore, consumption must change more in response to a TFP shock, which in turn increases its volatility. Sluggish capital also affects sectoral labor supply because constant capital in the short run implies that moving labor in and out of modern agriculture and nonagriculture becomes less advantageous. This implies, in turn, that aggregate labor move less in response to shocks to $z^M$ and $z^{AM}$ and more to shocks to $z^S$.

Panel B of Table 4 shows the results. For convenience, the statistics for the benchmark economy are restated as panel A of this table. The main effect of introducing adjustment costs is that the cyclical behavior of aggregate labor supply and consumption is more in line with the data: $n$ becomes negatively correlated with GDP and positively correlated with nonagricultural labor supply, as it is for China (see panel A of Table 2). Moreover, consumption becomes substantially more volatile, more negatively correlated with agricultural labor supply, and get a more reasonable correlation with GDP. However, aggregate investment is too smooth in this adjustment-cost experiment.

As the structural change progresses, the correlation between $n$ and GDP now increase more, while the initial fall in employment volatility becomes more pronounced, see panels C and D of Figure 7.3 in the appendix.

Table 3: Data vs Model with Lower Variance in Traditional Agr.: HP-filtered

| | $c$ | $i$ | $\frac{P^G y^G}{P}$ | $\frac{P^M y^M}{P}$ | $\frac{APL^G}{APL^M}$ | $n^G$ | $n^M$ | $n$ |
|---|---|---|---|---|---|---|---|---|
| | A. Benchmark Economy | | | | | | | $std\,(y) = 1.7\%$ |
| $\frac{std(x)}{std(y)}$ | 0.27 | 2.39 | 1.09 | 1.18 | 0.62 | 1.03 | 1.07 | 0.42 |
| $corr\,(x, y)$ | 0.81 | 0.99 | 0.30 | 0.97 | -0.38 | -0.25 | 0.73 | 0.43 |
| $corr\left(x, n^G\right)$ | -0.08 | -0.25 | 0.78 | -0.43 | 0.73 | 1 | -0.75 | 0.69 |
| $corr\left(x, n^M\right)$ | 0.45 | 0.75 | -0.31 | 0.87 | -0.74 | -0.75 | 1 | -0.21 |
| | B. Capital Adjustment Cost | | | | | | | $std\,(y) = 1.7\%$ |
| $\frac{std(x)}{std(y)}$ | 0.71 | 1.47 | 1.22 | 1.22 | 0.71 | 1.35 | 1.05 | 0.45 |
| $corr\,(x, y)$ | 0.99 | 0.99 | 0.14 | 0.96 | -0.41 | -0.49 | 0.58 | -0.27 |
| $corr\left(x, n^G\right)$ | -0.45 | -0.49 | 0.74 | -0.68 | 0.78 | 1 | -0.92 | 0.93 |
| $corr\left(x, n^M\right)$ | 0.53 | 0.77 | -0.09 | 0.88 | -0.50 | -0.56 | 1 | 0.14 |
| | C. Same autocorrelation for $Z^S$ and $Z^{AM}$ | | | | | | | $std\,(y) = 1.6\%$ |
| $\frac{std(x)}{std(y)}$ | 0.27 | 2.40 | 0.97 | 1.18 | 0.38 | 0.82 | 0.97 | 0.32 |
| $corr\,(x, y)$ | 0.80 | 0.99 | 0.33 | 0.98 | -0.68 | -0.34 | 0.83 | 0.54 |
| $corr\left(x, n^G\right)$ | -0.15 | -0.35 | 0.72 | -0.48 | 0.51 | 1 | -0.70 | 0.50 |
| $corr\left(x, n^M\right)$ | 0.49 | 0.86 | -0.15 | 0.93 | -0.77 | -0.70 | 1 | 0.08 |
| | D. Cobb-Douglas Preferences w/Agric. Subsistence Level | | | | | | | $std\,(y) = 1.6\%$ |
| $\frac{std(x)}{std(y)}$ | 0.42 | 2.44 | 1.21 | 1.16 | 0 | 0.86 | 0.65 | 0.41 |
| $corr\,(x, y)$ | 0.80 | 0.99 | 0.41 | 0.99 | 0 | -0.08 | 0.92 | 0.73 |
| $corr\left(x, n^G\right)$ | -0.26 | -0.06 | 0.82 | -0.09 | 0 | 1 | -0.32 | 0.53 |
| $corr\left(x, n^M\right)$ | 0.73 | 0.93 | 0.13 | 0.93 | 0 | -0.32 | 1 | 0.52 |

Table 4: Robustness analysis, benchmark model versus alternative model. All statistics refer to HP-filtered simulated data for 1985-2012.

#### 5.4.2 Modifying the TFP process for traditional sector, $z^S$

In the benchmark model the persistence of shocks to $z^S$ is calibrated to be much lower than that of shocks to $z^{AM}$ ($\hat{\phi}^S = 0.42$ versus $\hat{\phi}^{AM} = 0.90$). We argued above that the transitory nature of shocks to traditional sector $z^S$ was the cause behind the falling volatility of aggregate employment in the initial phase of the transition (see the lower right panel of Figure 11).

Note that since neither traditional nor modern agricultural production are directly observed, measurement error in agricultural capital and employment will show up as movements in the two TFP levels. This will affect the estimated TFP processes for $z^S$ and $z^M$. To evaluate how the results change in response to modifications of the TFP process in traditional sector, we consider a sensitivity analysis where TFP shocks to traditional agriculture has the same persistence as shocks to modern agriculture ($\hat{\phi}^S = \hat{\phi}^{AM} = 0.9$). Moreover, we adjust the volatility of the innovations to $z^S$, $\sigma\left(\epsilon_t^S\right)$, so that the stationary variance of $z^S$ is kept constant. The results are shown in panel C of Table 4. The main effect of a higher persistence of shocks to $z^S$ is that the aggregate volatility of labor supply $n$ falls and the correlation between $n$ and GDP increase somewhat. Moreover, the relative volatility of employment is predicted to increase monotonically during structural change (see panel D of Figure 7.3 in the appendix). Lowering the standard deviation of innovations to $z^S$, $\sigma\left(\epsilon_t^S\right)$, has similar qualitative effects.

#### 5.4.3 A large subsistence level of agriculture

Herrendorf et al. (2013) emphasize the role of a large subsistence component in agriculture, i.e., a large $\bar{c}$, combined with a low elasticity of substitution between goods. They argue that these features are needed to account for structural change in the US after 1950, provided that the elasticity of substitution is restricted to be the same across services, manufacturing, and agriculture (see our discussion in Section 3.5). Recall that when allowing for a subsistence level in agricultural goods, estimated model requires a very small $\bar{c}$. Nevertheless, it is interesting to investigate how a large $\bar{c}$ would influence the structural change and the business-cycle properties of the economy. We therefore reestimate the non-homothetic version of our model when imposing $\varepsilon = 1$ and removing the traditional agricultural sector. This implies a very large subsistence level in agriculture: the estimated $\bar{c}$ amounts to 98% of agricultural consumption in 1985. We label this economy as the *subsistence economy.*

As can be expected from the analysis of Section 4, the model with a low $\varepsilon$ and a large $\bar{c}$ provides a poorer fit of China's structural change than does our benchmark model. After 1990 the subsistence economy implies a too large expenditure share of agriculture and too low real output share. Note that a higher $\varepsilon$, which the unrestricted model calls for, will ameliorate the model's predictions on these moments. The model also predicts a too large capital share in agriculture, especially 1985-2005, and a too low employment share of agriculture.

The business-cycle properties of the subsistence economy are presented in panel D of Table

4. Most of the qualitative properties of this economy are similar to those of the benchmark economy. However, the subsistence economy implies a substantially higher correlation between $n$ and GDP. Note also that absent a traditional agricultural sector, the aggregate production function for agricultural goods becomes a Cobb-Douglas. This implies that this economy cannot generate any movements in the productivity gap (see Proposition 3).

Both the subsistence economy and the benchmark economy were estimated using the empirical levels of the expenditure share and output share of agriculture in China, and are therefore tailored to match the overall change between 1985 and 2012. In Section 3.5 we emphasized that the changes in expenditure and output shares of agriculture are consistent with $\varepsilon > 1$ even at higher frequencies, including 1-year and 5-year changes. It is therefore interesting to examine what our business-cycle models would predict about implied elasticities for changes at higher frequencies.

When plotting the 5-year changes in output share against changes in expenditure share of agriculture in our benchmark model, the observations fall on a line with an upward slope of 0.72. This is slightly lower than the empirical regression line for 5-year changes in China (slope of 0.81, see Figure 13 in the appendix). According to equation (27), this model's implied regression line correctly recovers the $\varepsilon$ we assumed in the benchmark economy ($\varepsilon = 3.6$). A similar exercise for the subsistence economy yields a regression line with slope 0.5, implying $\varepsilon = 2$. Thus the combination of $\varepsilon = 1$ with a high $\bar{c}$ can to some extent stand in for a larger elasticity of substitution even at high frequencies, although the magnitude of this elasticity is quantitatively too small.

# 6 Conclusion

Business cycle fluctuations in countries undergoing structural transformation differ systematically from business cycles in industrialized countries: we document that countries with large declining agricultural sectors – including China – have aggregate employment fluctuations that are smooth and acyclical, while these countries experience volatile and procyclical reallocation of labor between agriculture and nonagriculture. We provide an unified theoretical framework for studying business cycle during structural change. The central aspect of the theory is the modernization of agriculture that occurs during the structural change: agriculture is becoming increasingly capital intensive and less labor intensive as a large traditional sector is crowded out. With a large traditional sector the expansion of manufacturing draws workers from traditional agriculture, ensuring smooth aggregate employment and large reallocation of workers between sectors. This process is driven by capital accumulation and differential productivity growth between agriculture and nonagriculture. We calibrate the model to China and show that the model is consistent both with China's structural transformation and with the business cycle properties of China. Moreover, the model is consistent with the changing business cycle properties as the economy goes from a poor economy with a large agricultural sector to a modern industrialized economy with negligible agricultural employment.

# 7 Appendix

Note: this appendix is preliminary and incomplete.

## 7.1 Construction of cross-country data for business-cycle facts

We now describe how we have constructed the data on business-cycle facts across countries.

The data for aggregate GDP, capital stocks, investment are from the World Development Indicators. The data for value added in agriculture and capital stocks in agriculture is from the FAO.

The data for sectoral employment comes from the International Labor Organization (ILO). The data set is constructed as follows. First, we use data from the labor force surveys, households surveys, official statistics, and population censuses. We exclude data from firm surveys. Second, we exclude data that are not representative of the whole country. In particular, we exclude data from some countries which report data that only cover the urban population. Third, if multiple sources exist for the same country and these data cover overlapping time periods, we merge (by chaining) the different sources provided that data in the overlapping time periods are small. However, if the differences are large across different sources, we only retain the most recent data source, provided that the sample period is at least 15 years. If the most recent data cover less than 15 years, we retain the less recent data series (provided the sample covers at least 15 years).

Fourth, if multiple sources exist for the same country and these data do not cover overlapping time periods, then we do not merge the data. Instead we retain only the most recent data, provided that the sample period is at least 15 years. Again, if the most recent data covers less than 15 years, we use less recent data, provided that the data cover at least 15 years. The country is dropped if there are no data series longer than 15 years.

The resulting data set covers the following countries and time periods.

For panels a and c in Figure 3 (aggregate employment and GDP), the sample comprises 68 countries. The sample time periods for each country are the following

| Country | Start | End | Country | Start | End |
|---|---|---|---|---|---|
| ALB | 1983 | 2015 | ITA | 1977 | 2015 |
| AUS | 1983 | 2015 | JAM | 1992 | 2015 |
| AUT | 1983 | 2015 | JPN | 1970 | 2015 |
| AZE | 1990 | 2015 | KOR | 1970 | 2015 |
| BEL | 1983 | 2015 | LTU | 1998 | 2015 |
| BGR | 2000 | 2015 | LUX | 1983 | 2015 |
| BHS | 1989 | 2015 | LVA | 1996 | 2015 |
| BRA | 1981 | 2014 | MDA | 1999 | 2015 |
| BRB | 1981 | 2015 | MEX | 1995 | 2015 |
| CAN | 1970 | 2012 | MLT | 2000 | 2015 |
| CHE | 1991 | 2015 | MMR | 1978 | 1994 |
| CHL | 1975 | 2015 | MYS | 1980 | 2015 |
| CHN | 1985 | 2012 | NLD | 1987 | 2015 |
| CRI | 1980 | 2013 | NOR | 1972 | 2015 |
| CUB | 1995 | 2014 | NZL | 1986 | 2015 |
| CYP | 1999 | 2015 | PAK | 1973 | 2008 |
| CZE | 1993 | 2015 | PAN | 1982 | 2015 |
| DEU | 1983 | 2015 | PHL | 1971 | 2015 |
| DNK | 1972 | 2015 | POL | 1999 | 2015 |
| DOM | 1996 | 2015 | PRI | 1970 | 2011 |
| EGY | 1989 | 2015 | PRT | 1974 | 2015 |
| ESP | 1970 | 2015 | PRY | 1997 | 2015 |
| EST | 1995 | 2015 | ROU | 1997 | 2015 |
| FIN | 1970 | 2015 | RUS | 1997 | 2015 |
| FRA | 1970 | 2015 | SLV | 1994 | 2015 |
| GBR | 1983 | 2015 | SVK | 1994 | 2015 |
| GRC | 1983 | 2015 | SVN | 1995 | 2015 |
| HKG | 1978 | 2008 | SWE | 1970 | 2015 |
| HND | 1990 | 2015 | THA | 1971 | 2015 |
| HUN | 1992 | 2015 | TTO | 1977 | 2015 |
| IDN | 1985 | 2015 | USA | 1970 | 2015 |
| IRL | 1983 | 2015 | VEN | 1975 | 2013 |
| ISL | 1991 | 2015 | ZAF | 2000 | 2015 |
| ISR | 1970 | 2015 | | | |

For panel b in Figure 3 (agricultural versus nonagricultural employment), the sample comprises 67 countries. The sample time periods for each country are the following,

| Country | Start | End | Country | Start | End |
|---|---|---|---|---|---|
| ALB | 1994 | 2014 | ITA | 1977 | 2015 |
| AUS | 1970 | 2015 | JAM | 1992 | 2015 |
| AUT | 1983 | 2015 | JPN | 1970 | 2015 |
| AZE | 1983 | 2015 | KOR | 1970 | 2015 |
| BEL | 1983 | 2015 | LTU | 1998 | 2015 |
| BGR | 2000 | 2015 | LUX | 1983 | 2015 |
| BHS | 1989 | 2011 | LVA | 1996 | 2015 |
| BRA | 1983 | 2014 | MDA | 1999 | 2015 |
| BRB | 1981 | 2015 | MEX | 1995 | 2015 |
| CAN | 1970 | 2012 | MLT | 2000 | 2015 |
| CHE | 1991 | 2015 | MMR | 1978 | 1994 |
| CHL | 1975 | 2015 | MYS | 1980 | 2015 |
| CHN | 1985 | 2012 | NLD | 1987 | 2015 |
| CRI | 1980 | 2013 | NOR | 1972 | 2015 |
| CUB | 1995 | 2014 | NZL | 1986 | 2015 |
| CYP | 1999 | 2015 | PAK | 1973 | 2008 |
| CZE | 1993 | 2015 | PAN | 1982 | 2015 |
| DEU | 1983 | 2015 | PHL | 1971 | 2015 |
| DNK | 1972 | 2015 | POL | 1999 | 2015 |
| DOM | 1996 | 2015 | PRI | 1970 | 2011 |
| EGY | 1989 | 2015 | PRT | 1974 | 2015 |
| ESP | 1970 | 2015 | PRY | 1997 | 2015 |
| EST | 1989 | 2015 | ROU | 1997 | 2015 |
| FIN | 1970 | 2015 | RUS | 1997 | 2015 |
| FRA | 1970 | 2015 | SLV | 1994 | 2015 |
| GBR | 1983 | 2015 | SVK | 1994 | 2015 |
| GRC | 1983 | 2015 | SVN | 1993 | 2015 |
| HKG | 1978 | 2008 | SWE | 1970 | 2015 |
| HND | 1990 | 2015 | THA | 1971 | 2015 |
| HUN | 1992 | 2015 | TTO | 1977 | 2010 |
| IDN | 1985 | 2015 | USA | 1970 | 2015 |
| IRL | 1983 | 2015 | VEN | 1975 | 2013 |
| ISL | 1991 | 2015 | ZAF | 2000 | 2015 |
| ISR | 1970 | 2015 | | | |

For panel d in Figure 3 (productivity gap versus nonagricultural employment), the sample comprises 63 countries. The sample time periods for each country are the following,

| Country | Start | End | Country | Start | End |
|---|---|---|---|---|---|
| ALB | 1994 | 2014 | JAM | 1993 | 2015 |
| AUS | 1990 | 2015 | JPN | 1970 | 2015 |
| AUT | 1983 | 2015 | KOR | 1970 | 2015 |
| AZE | 1990 | 2015 | LTU | 1998 | 2015 |
| BEL | 1995 | 2015 | LUX | 1995 | 2015 |
| BGR | 2000 | 2015 | LVA | 1996 | 2015 |
| BHS | 1989 | 2011 | MDA | 1999 | 2015 |
| BRA | 1981 | 2014 | MEX | 1995 | 2015 |
| BRB | 1990 | 2014 | MLT | 2000 | 2015 |
| CHE | 1991 | 2015 | MYS | 1980 | 2015 |
| CHL | 1975 | 2015 | NLD | 1987 | 2015 |
| CHN | 1985 | 2012 | NOR | 1972 | 2015 |
| CRI | 1980 | 2013 | NZL | 1986 | 2015 |
| CUB | 1995 | 2014 | PAK | 1973 | 2008 |
| CYP | 1999 | 2015 | PAN | 1982 | 2015 |
| CZE | 1993 | 2015 | PHL | 1971 | 2015 |
| DEU | 1991 | 2015 | POL | 1999 | 2015 |
| DNK | 1972 | 2015 | PRI | 1971 | 2011 |
| DOM | 1996 | 2015 | PRT | 1995 | 2015 |
| EGY | 1989 | 2015 | PRY | 1997 | 2015 |
| ESP | 1995 | 2015 | ROU | 1997 | 2015 |
| EST | 1995 | 2015 | RUS | 1997 | 2015 |
| FIN | 1975 | 2015 | SLV | 1994 | 2015 |
| FRA | 1970 | 2015 | SVK | 1995 | 2015 |
| GBR | 1990 | 2015 | SVN | 1995 | 2015 |
| GRC | 1995 | 2015 | SWE | 1980 | 2015 |
| HND | 1990 | 2015 | THA | 1971 | 2015 |
| HUN | 1995 | 2015 | TTO | 1984 | 2010 |
| IDN | 1987 | 2015 | USA | 1970 | 2015 |
| IRL | 1995 | 2015 | VEN | 1975 | 2013 |
| ISL | 1997 | 2015 | ZAF | 2000 | 2015 |
| ITA | 1990 | 2015 | | | |

Table 5: Summary Statistics for China data and Model: First-differenced

## 7.2 Additional tables

| | $c$ | $i$ | $\frac{P^G y^G}{P}$ | $\frac{P^M y^M}{P}$ | $\frac{APL^G}{APL^M}$ | $n^G$ | $n^M$ | $n$ |
|---|---|---|---|---|---|---|---|---|
| | A. FD- Filtered China Data, 1985-2012: $std\,(y) = 2.4\%$ | | | | | | | |
| $\frac{std(x)}{std(y)}$ | 1.27 | 3.34 | 1.82 | 1.31 | 2.39 | 1.00 | 0.76 | 0.33 |
| $corr(x, y)$ | 0.57 | 0.63 | 0.12 | 0.93 | -0.09 | -0.57 | 0.66 | -0.25 |
| $corr(x, n^G)$ | -0.74 | -0.34 | -0.38 | -0.38 | 0.35 | 1.00 | -0.50 | 0.71 |
| $corr\left(x, n^M\right)$ | 0.32 | 0.37 | 0.40 | 0.53 | -0.52 | -0.50 | 1 | 0.19 |
| | B. FD- Filtered Model, Homothetic model $std\,(y) = 2.7\%$ | | | | | | | |
| $\frac{std(x)}{std(y)}$ | 0.30 | 2.36 | 1.11 | 1.25 | 0.72 | 1.10 | 1.27 | 0.49 |
| $corr\,(x, y)$ | 0.80 | 0.99 | 0.24 | 0.95 | -0.42 | -0.30 | 0.69 | 0.18 |
| $corr\left(x, n^G\right)$ | -0.22 | -0.27 | 0.80 | -0.51 | 0.79 | 1 | -0.78 | 0.75 |
| $corr\left(x, n^M\right)$ | 0.55 | 0.66 | -0.40 | 0.88 | -0.81 | -0.78 | 1 | -0.52 |

## 7.3 Additional figures

## 7.4 Derivation of Equation 15

**Proof.** We prove here both Lemma 1 and its corollary. We start by deriving the expression in (15). The FOC (10) that yields the equalization of the marginal product of capital can be rewritten as

$$\frac{1-\kappa}{\kappa} = \varsigma \frac{\gamma}{1-\gamma} \frac{1-\beta}{1-\alpha} \left(\frac{Y^G}{Y^M}\right)^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{Y^{AM}}{Y^G}\right)^{\frac{\omega-1}{\omega}}.$$

Taking logarithms, and letting $\omega = \varepsilon$ yields

$$\ln(1-\kappa) - \ln\kappa = \ln\left(\varsigma \frac{\gamma}{1-\gamma} \frac{1-\beta}{1-\alpha}\right) + \frac{\varepsilon-1}{\varepsilon} \ln\left(\frac{y^G}{y^M}\right)$$

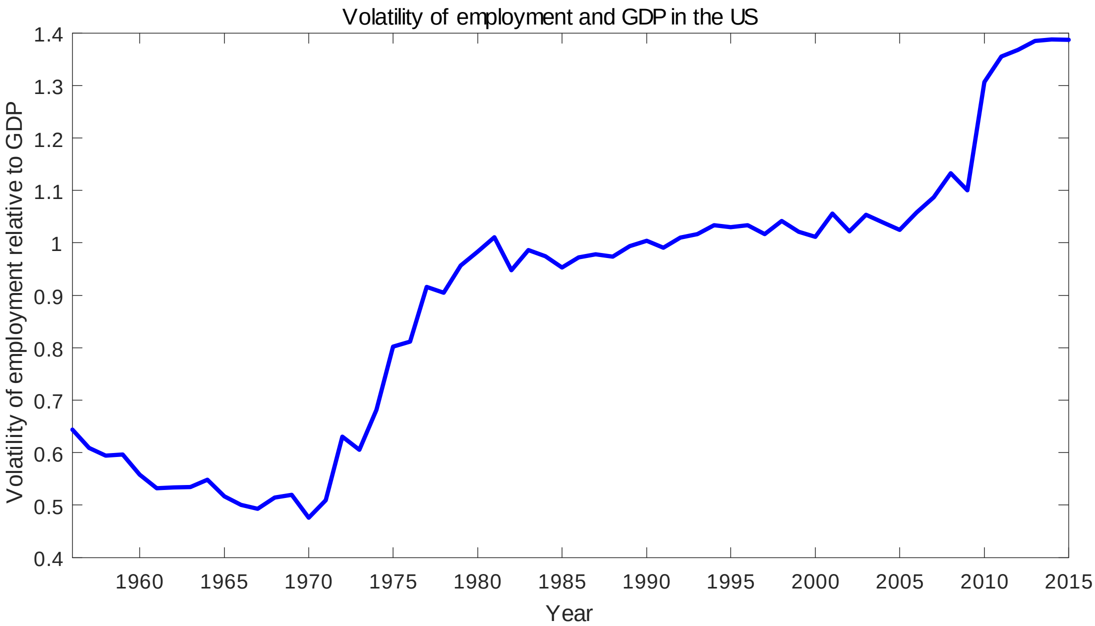



Figure 12: The figure shows the time evolution of the volatility of total employment in private sector (excluding government) relative to the GDP in the US from 1929 to 2015. The relative volatility is measured by the standard deviation of the HP-filtered log total employment divided by the HP-filtered log real output, both of which are computed on a 28-year rolling window. The x-axis denotes the end year of the sample window. The HP-filter use the smoothing parameter 6.25 (Ravn and Uhlig 2002). Source: Employment in private sectors is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. The GDP in current price is deflated by the implicit price deflators from NIPA Table 1.1.9.

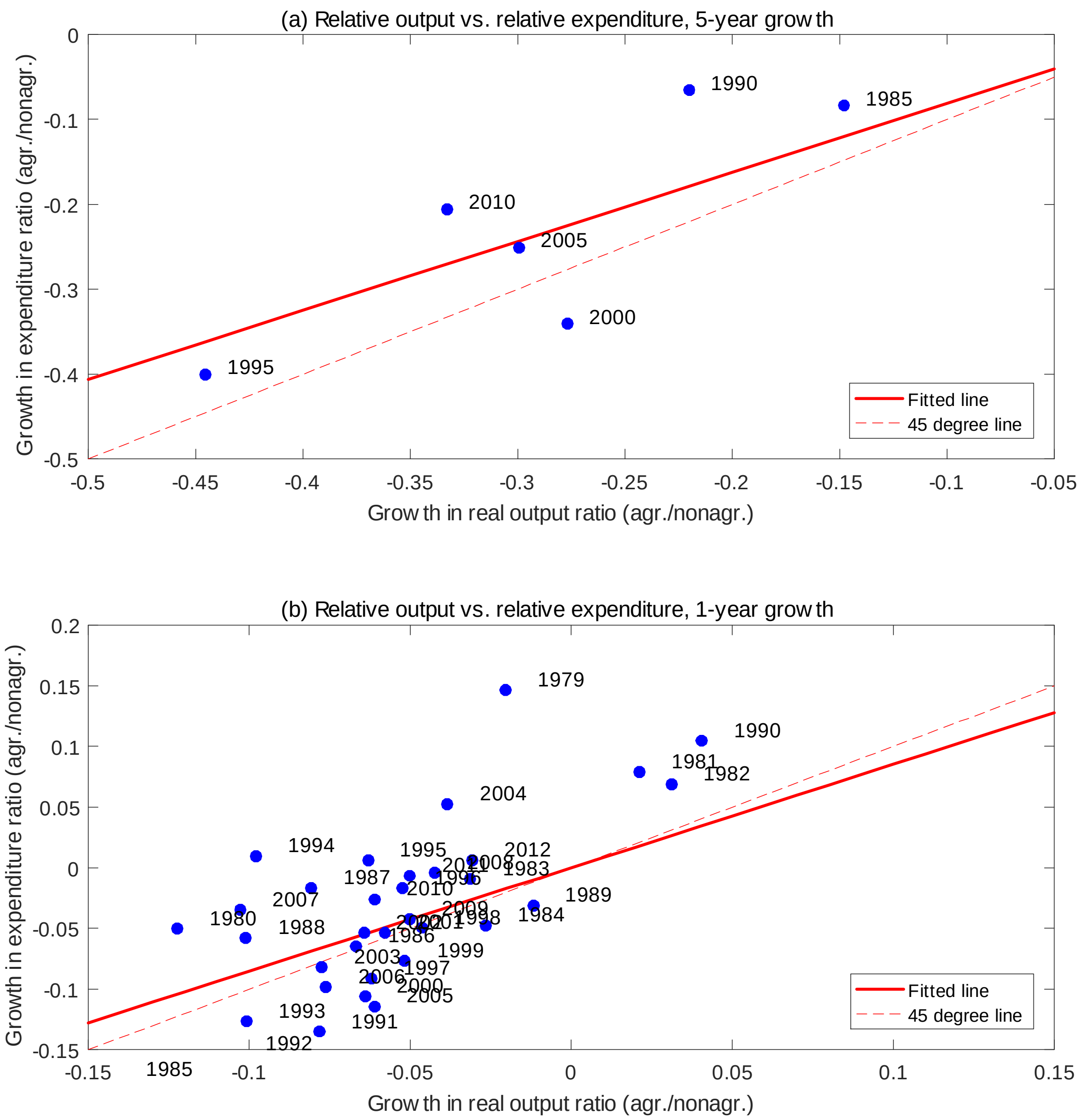
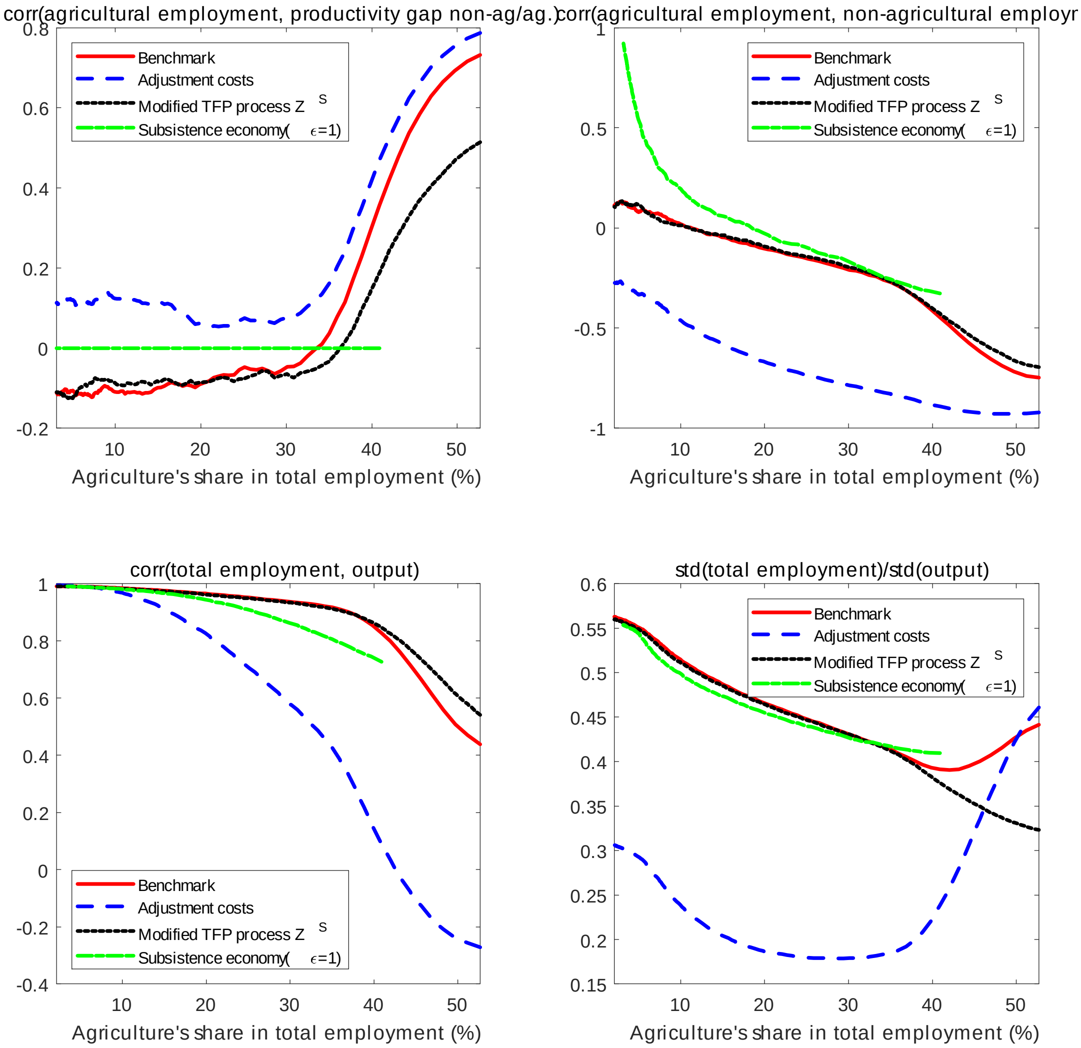



Figure 13: Panel a in the graph shows a scatter plot of 5-year changes in the log of the ratio of real agricultural to nonagricultural output against the log of the corresponding value added share (in current prices). The sample is 1978-2012 for China. An OLS regression without a constant term – in line with equation (27) – yields a slope of 0.8121 (std. err. 0.11, t=7.36, p=0.001), implying $\varepsilon \approx 5.2$. Similarly, Panel b in the graph shows a scatter plot of 1-year changes in the log of the ratio of real agricultural to nonagricultural output against the log of the corresponding value added share (in current prices). An OLS regression without a constant term yields a slope of 0.852 (std. err. 0.132, t=6.41, p=0.000), implying $\varepsilon \approx 6.75$.


corr(agricultural employment, productivity gap non-ag/ag.)
corr(agricultural employment, non-agricultural employr
corr(total employment, output)
std(total employment)/std(output)
Benchmark
Adjustment costs
Modified TFP process Z S
Subsistence economy( ϵ=1)
Agriculture's share in total employment (%)
10
20
30
40
50
0.8
0.6
0.4
0.2
0
-0.2
1
0.5
-0.5
-1
-0.4
0.55
0.45
0.35
0.3
0.25
0.15

Substituting in the expressions for $y^G$ and $y^G$, and differentiating with respect to $\ln\chi$ yields

$$\left(\frac{\varepsilon-1}{\varepsilon}(\beta-\alpha)-\frac{1}{\varepsilon}\frac{1}{1-\kappa}\right)\frac{\partial\ln\kappa}{\partial\ln\chi}=-\frac{\varepsilon-1}{\varepsilon}(\beta-\alpha)\times\left(1-\frac{\partial\ln\nu^M}{\partial\ln\chi}\right) \tag{30}$$

Next, consider (13). Differentiating with respect to $\ln\chi$ yields

$$\begin{aligned}\frac{\partial\ln\left(\nu^M\right)}{\partial\ln\chi} &= \frac{\nu^{AM}+(1+(1-\kappa)(\varepsilon-1)(1-\beta))\,\nu^S}{(1+\nu^S(1-\beta)(\omega-1))}\frac{1}{1-\kappa}\frac{\partial\ln\kappa}{\partial\ln\chi} \\ &\quad+\frac{\nu^S(\omega-1)(1-\beta)}{1+\nu^S(1-\beta)(\varepsilon-1)}\end{aligned} \tag{31}$$

Plugging-in (31) into (15) and simplifying terms leads to:

$$\frac{\partial\ln\kappa}{\partial\ln K}=\frac{(\varepsilon-1)(\beta-\alpha)(1-\kappa)}{1+(\varepsilon-1)\left((\beta-\alpha)\left(\kappa-\nu^M\right)+\nu^S(1-\beta)\right)}>0. \tag{32}$$

■

## 7.5 Proof of Lemma 2

Having defined $z_S\equiv Z^S/\left(Z^M\right)^\alpha$ and $z_A\equiv\left(Z^{AM}\right)^\beta/\left(Z^M\right)^\alpha$, the four static equilibrium conditions and the definition of $\Xi$ can be expressed as

$$\frac{\nu^S}{\nu^{AM}} = \left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^\varepsilon\left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\Xi\right)^{-(1-\beta)(\varepsilon-1)}\left(\frac{z_S}{z_A}\right)^{\varepsilon-1} \tag{33}$$

$$\frac{\nu^{AM}}{\nu^M} = \frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{1-\kappa}{\kappa} \tag{34}$$

$$\left(\frac{\nu^{AM}}{\nu^M}+\frac{\nu^S}{\nu^M}\right)\kappa\chi = \left(\frac{1}{\nu^M}-1\right)\kappa\chi=\Xi-\kappa\chi \tag{35}$$

$$\left(\frac{1-\kappa}{\kappa}\right)(\Xi)^{(\beta-\alpha)(\varepsilon-1)} = \left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^\varepsilon\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{\beta(\varepsilon-1)}(z_A)^{(\varepsilon-1)} \tag{36}$$

$$\Xi \equiv \frac{\kappa\chi}{\nu^M}=\left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\right)^{-1}\frac{(1-\kappa)\,\chi}{\nu^{AM}} \tag{37}$$

We start with the comparative statics for $z_A$. Rewrite eq. (36) as

$$\Xi=\left(\left(\frac{1-\kappa}{\kappa}\right)^{-1}\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{\beta(\varepsilon-1)}(z_A)^{(\varepsilon-1)}\right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}} \tag{38}$$

Substitute (33)-(34) into (35) to get rid of the ratios $\frac{\nu^S}{\nu^{AM}}$ and $\frac{\nu^{AM}}{\nu^M}$ and obtain an equation in $\kappa$ and $\Xi$;

$$\frac{\Xi}{\kappa\chi}-1=\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{1-\kappa}{\kappa}+\left(\frac{1-\varsigma}{\alpha}\frac{\gamma}{1-\gamma}\right)^{\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{(\varepsilon-1)(\beta-\alpha)}(\Xi)^{-(\varepsilon-1)(1-\alpha)}(z_S)^{\varepsilon-1}$$

Substitute in eq. (38), and simplify

$$\begin{aligned}
&\frac{1}{\chi}\left(\frac{\kappa}{1-\kappa}\right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}}\left(\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{\beta(\varepsilon-1)}\right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}}(z_A)^{\frac{1}{\beta-\alpha}}\\
=\;&\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}-\frac{\beta-\alpha}{\alpha(1-\beta)}\kappa+\left(\frac{1-\varsigma}{\alpha}\frac{\gamma}{1-\gamma}\right)^{\varepsilon}\left(\varsigma\frac{\gamma}{1-\gamma}\frac{\beta}{\alpha}\right)^{-\frac{(1-\alpha)}{(\beta-\alpha)}\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\frac{(1-\alpha)}{(\beta-\alpha)}(\beta(\varepsilon-1)-\varepsilon)+(\varepsilon-1)(\beta-\alpha)}\\
&\times(1-\kappa)^{\frac{(1-\alpha)}{(\beta-\alpha)}}(\kappa)^{-\frac{1-\beta}{\beta-\alpha}}(z_A)^{-(\varepsilon-1)\frac{(1-\alpha)}{(\beta-\alpha)}}(z_S)^{\varepsilon-1}
\end{aligned}$$

The LHS is increasing in $z_A$ and in $\kappa$. The RHS is decreasing in $z_A$ and in $\kappa$. It follows that an increasing in $z_A$ must be associated with a decline in $\kappa$. Equation (34) then implies that $\nu^{AM}/\nu^M$ increases.

Suppose $\partial\ln\Xi/\partial\ln z_A\geq 0$. Then the RHS of eq. (33) unambiguously falls, implying that the ratio $\frac{\nu^S}{\nu^{AM}}$ must also fall. Suppose instead that $\partial\ln\Xi/\partial\ln z_A<0$. Combine eq. (34) with the identity $1-\nu^S=\nu^M+\nu^{AM}$,

$$\begin{aligned}
1-\nu^S&=\left(1+\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{\kappa}{1-\kappa}\right)\nu^{AM}\\
&=\chi\left(\frac{1}{\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}}-\frac{\beta-\alpha}{\alpha(1-\beta)}\kappa\right)\frac{1}{\Xi}
\end{aligned}$$

Since $\partial\ln\kappa/\partial\ln z_A<0$ and since $\partial\ln\Xi/\partial\ln z_A<0$ by assumption, the RHS must increase.

It follows that $\nu^S$ must fall, implying that the ratio $\nu^S/\nu^{AM}$ must also fall. This proves that $\partial \ln \left(\nu^S/\nu^{AM}\right)/\partial \ln z_A < 0$ . It follows immediately that both $\upsilon$ and $\nu^{AM}$ must increase.

Consider now the comparative statics for $z_A$. Substitute (33)-(34) into (35) to get rid of the ratios $\frac{\nu^S}{\nu^{AM}}$ and $\frac{\nu^{AM}}{\nu^M}$ and obtain an equation in $\kappa$ and $\Xi$;

$$\frac{1}{1-\kappa} = \frac{\chi}{\Xi-\chi}\left[\left(1+\left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^{\varepsilon}\left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\Xi\right)^{-(1-\beta)(\varepsilon-1)}\left(\frac{z_S}{z_A}\right)^{\varepsilon-1}\right)\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}-1\right]$$

Rewrite (36) as,

$$\frac{1}{1-\kappa} = \left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{-\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\beta(\varepsilon-1)}(z_A)^{-(\varepsilon-1)}(\Xi)^{(\beta-\alpha)(\varepsilon-1)}+1.$$

Equate these expressions and rearrange to get one equation in $\Xi$;

$$\begin{aligned}
&\ln\left(\left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^{\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{(1-\beta)(\varepsilon-1)+1}(\Xi)^{-(1-\beta)(\varepsilon-1)}(z_S)^{\varepsilon-1}+\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}-\frac{\Xi}{\chi}\right)(z_A)^{\varepsilon-1}\right) \\
= \; &\ln\left(\frac{1}{\chi}\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{-\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\beta(\varepsilon-1)}\right)+(\beta-\alpha)(\varepsilon-1)\ln(\Xi)+\ln(\Xi-\chi) \qquad (39)
\end{aligned}$$

Differentiate equation (39) w.r.t. $z_S$, and rearranging terms yields

$$\begin{aligned}
&\frac{(\varepsilon-1)\left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^{\varepsilon}\left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\right)^{-(1-\beta)(\varepsilon-1)-1}(\Xi)^{-(1-\beta)(\varepsilon-1)}\left(\frac{z_S}{z_A}\right)^{\varepsilon-1}}{\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{-\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\beta(\varepsilon-1)}(z_A)^{-(\varepsilon-1)}(\Xi)^{(\beta-\alpha)(\varepsilon-1)}\left(\frac{\Xi-\chi}{\chi}\right)} \\
= \; &\left[(\beta-\alpha)(\varepsilon-1)+\frac{\Xi}{\Xi-\chi}+\frac{(1-\beta)(\varepsilon-1)\left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^{\varepsilon}\left(\frac{z_S}{z_A}\right)^{\varepsilon-1}\left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\right)^{-(1-\beta)(\varepsilon-1)-1}(\Xi)^{-(1-\beta)(\varepsilon-1)}+\frac{\Xi}{\chi}}{\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{-\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\beta(\varepsilon-1)}(z_A)^{-(\varepsilon-1)}(\Xi)^{(\beta-\alpha)(\varepsilon-1)}\left(\frac{\Xi-\chi}{\chi}\right)}\right.
\end{aligned}$$

Recall that $\Xi = \frac{\kappa\chi}{\nu^M} > \chi$ due to $\kappa > \nu^M$. Therefore both coefficient on $\partial\ln\Xi/\partial\ln z_S$ is positive. It follows that $\partial\ln\Xi/\partial\ln z_S > 0$. Now take log on both sides of equation (36) and differentiate w.r.t. $\ln z_S$; $0 = \partial/\partial\ln z_S \ln\left[(1-\kappa)/\kappa\right] + (\beta-\alpha)(\varepsilon-1)\partial\ln\Xi/\partial\ln z_S$. Since $(\beta-\alpha)(\varepsilon-1)\partial\ln\Xi/\partial\ln z_S > 0$, it must be that $\partial/\partial\ln z_S \ln\left[(1-\kappa)/\kappa\right] < 0$, which in turn implies that $\partial/\partial\ln z_S \ln\kappa > 0$. Recall that $\nu^{AM} = \chi(1-\kappa)/\Xi$. Since both $\Xi$ and $\kappa$ are increasing in $z_S$, $\nu^{AM}$ must be falling in $z_S$. Now substitute (34) into (35) to obtain an equation in $\frac{\nu^S}{\nu^{AM}}$, $\kappa$ and $\Xi$;

$$\left(\frac{\Xi}{\chi}-1\right)\frac{1}{1-\kappa}=\left(1+\frac{\nu^S}{\nu^{AM}}\right)\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}-1$$

Since both $\kappa$ and $\Xi$ are increasing in $z_S$, it follows that the ratio $\frac{\nu^S}{\nu^{AM}}$ must also be increasing in $z_S$. Since $\nu^S/\nu^{AM}=\beta^{-1}(1-\upsilon)/\upsilon$, it follows immediately that $\upsilon$ must fall in $z_S$.

## 7.6 Proof of Proposition 3

**Proof.** We start by evaluating Equations (17)-(18) under the ABGP conditions. Note that (14) implies that $\eta(1,1)=(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}$. Thus,

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= g_M=(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}(1-\alpha)\left(\frac{\chi}{Z^M}\right)^{-\alpha}-\delta-\rho,\\ \frac{\dot{\chi}_t}{\chi_t} &= g_M=(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}\times\left(\frac{\chi}{Z^M}\right)^{-\alpha}-(\delta+n)-\frac{c}{\chi}.\end{aligned}$$

Solving for $c/\chi$ and $\chi/Z^M$ yields the expressions in (21) and (22). Therefore, (17)-(18) hold true under the ABGP conditions. It is straightforward to see that under the ABGP conditions (in particular, when $\kappa=\upsilon=1$) (19)-(20) yields $\frac{\dot{\kappa}}{\kappa}=\frac{\dot{\upsilon}}{\upsilon}=0$. Likewise, (10) holds true when $\kappa=\upsilon=1$.

Next, consider the asymptotic growth rates of the sectoral capital. Taking logarithms and differentiating with respect to time the definitions of $k^M$ and $k^{AM}$ yields $\dot{k}^M/k^M=\dot{\kappa}/\kappa+\dot{\chi}/\chi=g^M$ and $\dot{k}^{AM}/k^{AM}=-(1-\kappa)^{-1}\times\dot{\kappa}/\kappa+\dot{\chi}/\chi=g^M-(\varepsilon-1)\beta\left(g^M-g^{AM}\right)$.

Next, consider the asymptotic growth rates of the sectoral employments of labor. First, observe that Equation (13) yields $\nu^M=1$ at the ABGP conditions $\kappa=\upsilon=1$. Second, note that $\nu^M=1$ implies that $\dot{N}^M/N^M=\dot{N}/N=n$. In order to establish the growth rate of $N^{AM}$, observe that taking logarithms on both side of Equation (11), differentiating with respect to time, and using the ABGP conditions and Equation (19) yields

$$\frac{\dot{N}^{AM}}{N^{AM}}=-\frac{1}{1-\kappa}\frac{\dot{\kappa}}{\kappa}+\frac{\dot{N}^{AM}}{N^{AM}}=n-(\varepsilon-1)\beta\left(g^M-g^{AM}\right).$$

Finally, to establish the growth rate of $N^S$, observe that taking logarithms on both side of

Equation (12), differentiating with respect to time, and using the ABGP conditions and Equation (20) yields

$$\frac{\dot{N}^S}{N^S} = -\frac{1}{1-\upsilon}\frac{\dot{\upsilon}}{\upsilon} + \frac{\dot{N}^{AM}}{N^{AM}} = \frac{\dot{N}^{AM}}{N^{AM}} - (\omega-1)\left[\left(g^{AM}-g^S\right) + (1-\beta)\left(g^M - g^{AM}\right)\right].$$

To establish convergence, we linearize the dynamic system in a neighborhood of the ABGP. The system has three predetermined variables $(\chi, \kappa, \upsilon)$ and one jump variable $(c)$. Therefore, we must prove that the linear system has three negative eigenvalues and one positive eigenvalue. The rest of the proof is devoted to establish that this is the case.

Let $\tilde{\chi} = \frac{\chi}{Z^M}$ and $\tilde{c} = \frac{c}{Z^M}$, implying that $\frac{d\tilde{\chi}/dt}{\tilde{\chi}} = \frac{\dot{\chi}}{\chi} - g^M$. Then, we can write the dynamic system (17)-(18)-(19)-(20). We can rewrite the system as ■

$$\begin{aligned}
\frac{d\tilde{c}/dt}{\tilde{c}} &= \eta\left(\kappa_t, \upsilon_t\right)^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^M\left(\kappa_t,\upsilon_t\right)}\right)^{-\alpha} - \delta - \rho - g^M \\
\frac{d\tilde{\chi}/dt}{\tilde{\chi}} &= \eta\left(\kappa_t, \upsilon_t\right)^{1-\alpha}\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^M\left(\kappa_t,\upsilon_t\right)}\right)^{-\alpha}\kappa_t - \delta - \tilde{c}_t/\tilde{\chi}_t - n - g^M \\
\frac{\dot{\kappa}}{\kappa} &= (1-\kappa)\frac{\begin{pmatrix}\left(\alpha g^M - \beta g^{AM} + (\beta-\alpha)\left(\frac{d\hat{\chi}/dt}{\hat{\chi}} + g^M\right)\right) \\ +\left(\frac{1}{\omega-1} - \frac{(\beta-\alpha)\left(1-\nu^M\right)}{1-\upsilon(1-\beta)}\right)\frac{\dot{\upsilon}}{\upsilon}\end{pmatrix}}{\frac{1}{\varepsilon-1} + (\beta-\alpha)\left(\kappa-\nu^M\right)} \\
\frac{\dot{\upsilon}}{\upsilon} &= (1-\upsilon)\frac{\begin{pmatrix}\beta g^{AM} - g^S + \\ (1-\beta)\left(\frac{\dot{\chi}_t}{\chi_t} - \frac{\dot{\kappa}_t}{\kappa_t}\frac{\kappa_t - \nu^M(\kappa_t,\upsilon_t)}{1-\kappa_t}\right)\end{pmatrix}}{\frac{1}{\omega-1} + \frac{(1-\upsilon)(1-\beta)(1-\nu^M)}{1-\upsilon(1-\beta)}}
\end{aligned}$$

where we use Equation (13) implying that

$$\frac{1-\nu^M}{\nu^M} = \frac{1-\kappa}{\kappa}\frac{1-\alpha}{1-\beta}\left(\frac{\beta}{\alpha} + \frac{1}{\alpha}\frac{1-\upsilon}{\upsilon}\right).$$

The transversality condition (TVC) becomes

$$\lim_{t\to\infty} \xi e^{-(\rho-n)t} K = 0$$

Substitute the condition that

$$\lim_{t\to\infty} \frac{\dot{K}}{K} = n + g^M$$

The TVC becomes

$$\lim_{t\to\infty} \frac{\dot{\xi}}{\xi} + g^M + n < \rho - n$$

Because

$$-\frac{\dot{c}}{c} = (1-\theta)(1-\sigma)\frac{\dot{h}}{h}\frac{h}{1-h} + \frac{\dot{\xi}}{\xi} + n$$

Then we have

$$\lim_{t\to\infty} \frac{\dot{c}}{c} = -\lim_{t\to\infty} \frac{\dot{\xi}}{\xi} - n$$

where we use $\lim_{t\to\infty} \frac{\dot{h}}{h} = 0$. Plug $\lim_{t\to\infty} \frac{\dot{\xi}}{\xi} = -g^M - n$ into the TVC to get

$$-g^M - n + g^M + n < \rho - n$$

In the end, the TVC can be rewritten as

$$\rho - n > 0$$

Letting $\Psi = (\tilde{c}, \tilde{\chi}, \kappa, \upsilon)'$, we can write the system of differential equations as

$$A\dot{\Psi} = f(\Psi)$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & a & 1 & b \\ 0 & c & d & 1 \end{bmatrix}$$

and, after

$$
\begin{aligned}
a &= -\frac{(1-\kappa)(\beta-\alpha)}{\frac{1}{\varepsilon-1}+(\beta-\alpha)\left(\kappa-\nu^{M}\right)} \\
b &= -(1-\kappa)\frac{\left(\frac{1}{\omega-1}-\frac{(\beta-\alpha)\left(1-\nu^{M}\right)}{1-\upsilon(1-\beta)}\right)}{\frac{1}{\varepsilon-1}+(\beta-\alpha)\left(\kappa-\nu^{M}\right)} \\
c &= -\frac{(1-\upsilon)(\omega-1)(1-\beta)}{\left(1+\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}(\omega-1)\nu^{M}\right)} \\
d &= -\frac{(1-\upsilon)(\omega-1)(1-\beta)\left(1-\frac{1-\alpha}{(1-\beta)\alpha}\frac{\nu^{M}}{\kappa}\left(\beta+\frac{1-\upsilon}{\upsilon}\right)\right)}{1+(\omega-1)\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}\nu^{M}}
\end{aligned}
$$

$$
f=\begin{bmatrix}
\eta\left(\kappa_t,\upsilon_t\right)^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^{M}(\kappa_t,\upsilon_t)}\right)^{-\alpha}-\delta-\rho-g^{M} \\
\eta\left(\kappa_t,\upsilon_t\right)\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^{M}(\kappa_t,\upsilon_t)}\right)^{-\alpha}\kappa_t-\delta-\tilde{c}_t/\tilde{\chi}_t-n-g^{M} \\
(1-\kappa)\frac{\beta\left(g^{M}-g^{AM}\right)}{\frac{1}{\varepsilon-1}+(\beta-\alpha)(\kappa-\nu^{M})} \\
\frac{(1-\upsilon)(\omega-1)\left(g^{AM}-g^{S}+(1-\beta)\left(g^{M}-g^{AM}\right)\right)}{1+\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}\nu^{M}(\omega-1)}
\end{bmatrix}
$$

where we use the

$$
\begin{aligned}
\frac{1-\nu^{M}}{\nu^{M}} &= \frac{1-\kappa}{\kappa}\frac{1-\alpha}{(1-\beta)}\left(\frac{\beta}{\alpha}+\frac{1}{\alpha}\frac{1-\upsilon}{\upsilon}\right) \\
\frac{1-\nu^{M}}{1-\upsilon(1-\beta)} &= \nu^{M}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{(1-\beta)}\frac{1}{\alpha\upsilon} \\
\frac{\kappa-\nu^{M}}{1-\kappa} &= -1+\frac{\nu^{M}}{\kappa}\frac{1-\alpha}{(1-\beta)\alpha}\left(\beta+\frac{1-\upsilon}{\upsilon}\right)
\end{aligned}
$$

The inverse of matrix $A$ is given by

$$
A^{-1}=\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & \frac{1}{bd-1}(a-bc) & -\frac{1}{bd-1} & \frac{b}{bd-1} \\
0 & \frac{1}{bd-1}(c-ad) & \frac{d}{bd-1} & -\frac{1}{bd-1}
\end{bmatrix}
$$

Along the approximate balanced growth path,

$$a^* = 0, b^* = 0, c^* = 0, d^* = 0$$

Now we compute

$$J(\Psi) = \begin{pmatrix} J_1(\Psi) \\ J_2(\Psi) \\ J_3(\Psi) \\ J_4(\Psi) \end{pmatrix} = A^{-1}(\Psi) f(\Psi)$$

where

$$\begin{aligned}
J_1(\Psi) &= \eta^{\frac{1}{\varepsilon}} (1-\gamma)(1-\alpha) \left( \frac{\kappa \tilde{\chi}}{\nu^M(\kappa, \upsilon)} \right)^{-\alpha} - \delta - \rho - g^M \\
J_2(\Psi) &= \eta \left( \frac{\kappa \tilde{\chi}}{\nu^M(\kappa, \upsilon)} \right)^{-\alpha} \kappa - \delta - \tilde{c}(\tilde{\chi})^{-1} - n - g^M \\
J_3(\Psi) &= \frac{1}{bd-1}(a-bc) \left( \left( \frac{\kappa \tilde{\chi}}{\nu^M(\kappa, \upsilon)} \right)^{-\alpha} \kappa - \delta - \tilde{c}/\tilde{\chi} - n - g^M \right) \\
&\quad - \frac{1}{bd-1}(1-\kappa) \frac{\beta\left(g^M - g^{AM}\right)}{\frac{1}{\varepsilon-1} + (\beta-\alpha)(\kappa - \nu^M)} \\
&\quad + \frac{b}{bd-1} \frac{(1-\upsilon)(\omega-1)\left(g^{AM} - g^S + \left(g^M - g^{AM}\right)(1-\beta)\right)}{1 + \frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}(\omega-1)\nu^M} \\
J_4(\Psi) &= \frac{1}{bd-1}(c-ad) \left( \left( \frac{\kappa \tilde{\chi}}{\nu^M(\kappa, \upsilon)} \right)^{-\alpha} \kappa - \delta - \tilde{c}/\tilde{\chi} - n - g^M \right) \\
&\quad + \frac{d}{bd-1}(1-\kappa) \frac{\beta\left(g^M - g^{AM}\right)}{\frac{1}{\varepsilon-1} + (\beta-\alpha)(\kappa - \nu^M)} \\
&\quad - \frac{1}{bd-1} \frac{(1-\upsilon)(\omega-1)\left(g^{AM} - g^S + \left(g^M - g^{AM}\right)(1-\beta)\right)}{1 + \frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}(\omega-1)\nu^M}
\end{aligned}$$

From (13) it follows that

$$\begin{aligned}
\dot{\nu}^M &= \nu^M \left( \frac{\dot{\kappa}}{\kappa} \frac{1-\nu^M}{1-\kappa} + \frac{\dot{\upsilon}}{\upsilon} \frac{1-\nu^M}{1-\upsilon\,(1-\beta)} \right) \\
&= \nu^M \left( \frac{\dot{\kappa}}{\kappa} \frac{\nu^M}{\kappa} \frac{1-\alpha}{(1-\beta)} \left( \frac{\beta}{\alpha} + \frac{1}{\alpha} \frac{1-\upsilon}{\upsilon} \right) + \frac{\upsilon}{\upsilon} \frac{1-\nu^M}{1-\upsilon\,(1-\beta)} \right)
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{\partial \nu^M}{\partial \kappa} &= \left( \frac{\nu^M}{\kappa} \right)^2 \frac{1-\alpha}{(1-\beta)} \left( \frac{\beta}{\alpha} + \frac{1}{\alpha} \frac{1-\upsilon}{\upsilon} \right) \\
\frac{\partial \nu^M}{\partial \upsilon} &= \frac{\nu^M}{\upsilon} \frac{1-\nu^M}{1-\upsilon\,(1-\beta)}.
\end{aligned}$$

Computing the Jacobian evaluated at the balanced growth path $(\tilde{c}, \tilde{\chi}, \kappa, \upsilon)'$, we obtain

$$J = \begin{bmatrix} 0 & J_{12}^* & J_{13}^* & 0 \\ J_{21}^* & J_{22}^* & J_{23}^* & 0 \\ 0 & 0 & J_{33}^* & 0 \\ 0 & 0 & J_{43}^* & J_{44}^* \end{bmatrix}$$

with determinant given by $-J_{12}^* J_{21}^* J_{33}^* J_{44}^*$ and four eigenvalues given by

$$\begin{bmatrix} \frac{1}{2} J_{22}^* + \frac{1}{2} \sqrt{\left(J_{22}^*\right)^2 + 4 J_{12}^* J_{21}^*} \\ \frac{1}{2} J_{22}^* - \frac{1}{2} \sqrt{\left(J_{22}^*\right)^2 + 4 J_{12}^* J_{21}^*} \\ J_{33}^* \\ J_{44}^* \end{bmatrix},$$

where

$$
\begin{aligned}
J_{12}^{*} &= -\alpha\left(1-\gamma\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(1-\alpha\right)\left(\tilde{\chi}^{*}\right)^{\alpha-1}<0 \\
J_{21}^{*} &= -\left(\tilde{\chi}^{*}\right)^{-1}<0 \\
J_{22}^{*} &= \left(\tilde{\chi}^{*}\right)^{-1}\left(\rho-n\right)>0 \\
J_{33}^{*} &= -\left(\varepsilon-1\right)\beta\left(g^{M}-g^{AM}\right)<0 \\
J_{44}^{*} &= -\left(\omega-1\right)\left(g^{AM}-g^{S}+\left(g^{M}-g^{AM}\right)\left(1-\beta\right)\right)<0
\end{aligned}
$$

Thus, three eigenvalues are negative, while one is positive ($J_{44}^{*}>0$), establishing the result.

# 8 Additional Material

## 8.1 The Equivalence between RCE and CSP

This section proves the equivalence between the recursive competitive equilibrium (RCE) and the constrained social planner's (CSP) problem. Without the loss of generality, we assume exogenous labor supply and there is no uncertainty. We prove by comparing the FOCs of two problems.

Denote the rental price of capital, land, and labor by $R, R^{L}, W^{i}, i=AM, M, S$, respectively. Consider the firms' problem in the nonagricultural sector

$$
\pi^{M}=\max_{H^{M},K^{M}} P^{M}Z^{M}\left(K^{M}\right)^{1-\alpha}\left(N^{M}\right)^{\alpha}-W^{M}H^{M}-RK^{M}
$$

where

$$
P^{M}=\left(1-\gamma\right)\left(\frac{Y^{M}}{Y}\right)^{-\frac{1}{\varepsilon}}
$$

and we normalized the price of final good in each period to 1. Therefore, we have

$$
\begin{aligned}
W^M &= (1-\gamma)\left(\frac{Y^M}{Y}\right)^{-\frac{1}{\varepsilon}} Z^M \alpha \left(K^M\right)^{1-\alpha} \left(N^M\right)^{\alpha-1} \\
R &= (1-\gamma)\left(\frac{Y^M}{Y}\right)^{-\frac{1}{\varepsilon}} Z^M (1-\alpha) \left(K^M\right)^{-\alpha} \left(N^M\right)^{\alpha}
\end{aligned}
$$

Similarly, we solve the problem of modern agricultural firms

$$
\pi^{AM} = \max_{H^{AM},K^{AM}} P^{AM} Z^{AM} \left(K^{AM}\right)^{(1-\beta)} \left(N^{AM}\right)^{\beta} - W^{AM} N^{AM} - rK^{AM} - R^L
$$

where

$$
P^{AM} = \gamma \left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}} \zeta \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}}
$$

and we must have

$$
\begin{aligned}
W^{AM} &= \gamma \left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}} \zeta \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} \beta Z^{AM} \left(K^{AM}\right)^{(1-\beta)} \left(N^{AM}\right)^{\beta-1} \\
R &= \gamma \left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}} \zeta \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} (1-\beta) Z^{AM} \left(K^{AM}\right)^{-\beta} \left(N^{AM}\right)^{\beta} \\
R^L &= \gamma \left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}} \zeta \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} (1-(1-\beta)-\beta) Z^{AM} \left(K^{AM}\right)^{(1-\beta)} \left(N^{AM}\right)^{\beta}
\end{aligned}
$$

and the problem of traditional agricultural worker

$$
\pi^S = \max_{H^S} P^S Z^S H^S - W^S N^S
$$

where

$$
P^S = \gamma \left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}} (1-\zeta) \left(\frac{Y^S}{Y^G}\right)^{-\frac{1}{\omega}}
$$

and we must have

$$
W^S = \gamma \left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}} \zeta \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} Z^S
$$

Second, we consider the households' problem (for simplicity, we assume exogenous labor supply here)

$$\max_{c,K} \int e^{-(\rho-n)t} \ln c \; dt$$

subject to

$$\dot{K} = N\left[W^{AM}\nu^{AM} + W^{M}(1-\tau)\left(1-\nu^{AM}-\nu^{S}\right) + W^{S}\nu^{S}\right] - \delta K + rK + R^{L} + Tr - cN$$

where

$$Tr = \tau W^{M}\left(1-\nu^{AM}-\nu^{S}\right)N$$

denotes the lump-sum transfer from the government to the households. The current value of Hamilton

$$H = \theta \ln c + \xi \left\{ \begin{array}{c} N\left[W^{AM}\nu^{AM} + W^{M}(1-\tau)\left(1-\nu^{AM}-\nu^{S}\right) + W^{S}\nu^{S}\right] \\ -\delta K + rK + R^{L} + Tr - cN \end{array} \right\}$$

The FOCs are given by

$$\begin{aligned} \frac{\theta}{c} &= \xi \\ \dot{\xi} - (\rho - n)\xi &= -\xi(r-\delta) \\ W^{AM} &= (1-\tau)W^{M} \\ W^{S} &= (1-\tau)W^{M} \end{aligned}$$

Therefore, we have

$$\begin{aligned} & W^{AM}\nu^{AM} + W^{S}\nu^{S} + W^{M}(1-\tau)\left(1-\nu^{AM}-\nu^{S}\right) \\ = \; & (1-\tau)W^{M} \\ = \; & (1-\tau)(1-\gamma)\left(\frac{Y^{M}}{Y}\right)^{-\frac{1}{\varepsilon}} Z^{M}\alpha\left(K^{M}\right)^{1-\alpha}\left(N^{M}\right)^{\alpha-1} \end{aligned}$$

The FOC w.r.t. $K$ becomes

$$\begin{aligned}\dot{\xi}-(\rho-n)\,\xi &= -\xi\left((1-\gamma)\left(\frac{Y^M}{Y}\right)^{-\frac{1}{\varepsilon}} Z^M(1-\alpha)\left(K^M\right)^{-\alpha}\left(N^M\right)^{\alpha}-\delta\right)\\ &= -\xi\left((1-\gamma)\,Y^{\frac{1}{\varepsilon}}\left(Y^M\right)^{1-\frac{1}{\varepsilon}}(1-\alpha)\,\frac{1}{K^M}-\delta\right)\end{aligned}$$

From the problem of constrained social planner's problem, we have the FOCs to be

$$\begin{aligned}\xi &: \dot{\xi}-(\rho-n)\,\xi\\ &= -\xi\left[Y^{\frac{1}{\varepsilon}}\frac{1}{K}\frac{1}{\kappa}(1-\gamma)\left(Y^M\right)^{1-\frac{1}{\varepsilon}}(1-\alpha)-\delta\right]\\ &= -\xi\left(Y^{\frac{1}{\varepsilon}}\frac{1}{K^M}(1-\gamma)\left(Y^M\right)^{1-\frac{1}{\varepsilon}}(1-\alpha)-\delta\right)\end{aligned}$$

The FOC w.r.t. $c$

$$c:\frac{\theta}{c}=\xi N$$

Therefore, we can find that the FOCs of two problems are the same.

We can further verify the resource constraints are the same. The households income becomes

$$\begin{aligned}& N\left[W^{AM}\nu^{AM}+W^M(1-\tau)\left(1-\nu^{AM}-\nu^S\right)+W^S\nu^S\right]+rK+R^L+Tr\\ =\;& NW^{AM}\nu^{AM}+rK^{AM}+R^L+W^M(1-\tau)\left(1-\nu^{AM}-\nu^S\right)+rK^M+Tr+W^S\nu^S N\\ =\;& Y\end{aligned}$$

Therefore,

$$\dot{K}=Y-\delta K-cN$$

## 8.2 Proof of Proposition 3.3

Recall that

$$\begin{aligned} P^{AM} &= \gamma\left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}}\zeta\left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} \\ P^M &= (1-\gamma)\left(\frac{Y^M}{Y}\right)^{-\frac{1}{\varepsilon}} \\ P^S &= \gamma\left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}}(1-\zeta)\left(\frac{Y^S}{Y^G}\right)^{-\frac{1}{\omega}} \end{aligned}$$

Thus, using the definition of $APL$ :

$$\begin{aligned} \frac{APL^M}{APL^G} &= \frac{(1-\gamma)\frac{\left(\frac{Y^M}{Y}\right)^{-\frac{1}{\varepsilon}}Y^M}{N^M}}{\frac{\gamma\left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}}\zeta\left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}}Y^{AM}+\gamma\left(\frac{Y^G}{Y}\right)^{-\frac{1}{\varepsilon}}(1-\zeta)\left(\frac{Y^S}{Y^G}\right)^{-\frac{1}{\omega}}Y^S}{N^{AM}+N^S}} \\ &= \frac{1-\gamma}{\gamma}\left(\frac{Y^M}{Y^G}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{\nu^{AM}+\nu^S}{\nu^M} \\ &= \frac{1}{1-\tau}\frac{1-\upsilon(1-\beta)}{\alpha} \end{aligned}$$

where the first inequality follows from the production function for $Y^G$ while the second follows from observing that the FOCs (11) and (12) imply that

$$\begin{aligned} \frac{\nu^{AM}}{\nu^M} &= \frac{\gamma\left(Y^G\right)^{1-\frac{1}{\varepsilon}}\upsilon}{(1-\tau)(1-\gamma)\left(Y^M\right)^{1-\frac{1}{\varepsilon}}}\frac{\beta}{\alpha}, \\ \frac{\nu^S}{\nu^M} &= \frac{\gamma\left(Y^G\right)^{1-\frac{1}{\varepsilon}}(1-\upsilon)}{(1-\tau)(1-\gamma)\left(Y^M\right)^{1-\frac{1}{\varepsilon}}}\frac{1}{\alpha}. \end{aligned}$$

# References


Acemoglu, D. and V. Guerrieri (2008, 06). Capital deepening and nonbalanced economic growth. *Journal of Political Economy 116*(3), 467–498.

Acemoglu, D. and F. Zilibotti (1997). Was prometheus unbound by chance? risk, diversification, and growth. *Journal of Political Economy 105*(4), 709–51.

Aguiar, M. and G. Gopinath (2007). Emerging market business cycles: The cycle is the trend. *Journal of Political Economy 115*, 69–102.

Alder, S., A. Mueller, and T. Boppart (2018). A theory of structural change that can fit the data. 2018 Meeting Papers 988, Society for Economic Dynamics.

Alvarez-Cuadrado, F., N. Long, and M. Poschke (2017, September). Capital-labor substitution, structural change and growth. *Theoretical Economics 12*(3).

Alvarez-Cuadrado, F. and M. Poschke (2011, July). Structural change out of agriculture: Labor push versus labor pull. *American Economic Journal: Macroeconomics 3*(3), 127–58.

Bai, C.-E. and Z. Qian (2010, December). The factor income distribution in China: 1978-2007. *China Economic Review 21*(4), 650–670.

Baumol, W. J. (1967). Macroeconomics of unbalanced growth: The anatomy of urban crisis. *The American Economic Review 57*(3), 415–426.

Benhabib, J., R. Rogerson, and R. Wright (1991, 02). Homework in macroeconomics: Household production and aggregate fluctuations. *Journal of Political Economy 99*, 1166–87.

Boppart, T. (2014). Structural change and the kaldor facts in a growth model with relative price effects and non-gorman preferences. *Econometrica 82*(6), 2167–2196.

Bouakez, H., E. Cardia, and F. J. Ruge-Murcia (2009). The transmission of monetary policy in a multisector economy. *International Economic Review 50*(4), 1243–1266.

Brandt, L. and X. Zhu (2010, Feb). Accounting for China's Growth. IZA Discussion Papers 4764, Institute for the Study of Labor (IZA).

Buera, F. and J. Kaboski (2009). Can traditional theories of structural change fit the data? *Journal of the European Economic Association 7*(2-3), 469–477.

Cass, D. (1965). Optimum growth in an aggregative model of capital accumulation. *The Review of Economic Studies 32*(3), 233–240.

Cheremukhin, A., M. Golosov, S. Guriev, and A. Tsyvinski (2017, 2). *The Economy of Peopleï£¡ï£¡s Republic of China from 1953.* Department of Economics, Yale University.

Christiano, L. J., M. Boldrin, and J. D. M. Fisher (2001, March). Habit Persistence, Asset Returns, and the Business Cycle. *American Economic Review 91*(1), 149–166.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Comin, D. A., D. Lashkari, and M. Mestieri (2015, September). Structural change with long-run income and price effects. Working Paper 21595, National Bureau of Economic Research.

Da Rocha, J. M. and D. Restuccia (2006, July). The Role of Agriculture in Aggregate Business Cycles. *Review of Economic Dynamics 9*(3), 455–482.

Herrendorf, B., R. Rogerson, and Ã. Valentinyi (2013, December). Two perspectives on preferences and structural transformation. *American Economic Review 103*(7), 2752–89.

Holz, C. (2006, 07). Measuring chinese productivity growth, 1952-2005.

Hornstein, A. (2000). The business cycle and industry comovement. *Economic Quarterly* (Win), 27–48.

Hornstein, A. and J. Praschnik (1997, December). Intermediate inputs and sectoral comovement in the business cycle. *Journal of Monetary Economics 40*(3), 573–595.

Horvath, M. (2000a). Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics 45*(1), 69 – 106.

Horvath, M. (2000b, February). Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics 45*(1), 69–106.

Iacoviello, M. and S. Neri (2010, April). Housing market spillovers: Evidence from an estimated dsge model. *American Economic Journal: Macroeconomics 2*(2), 125–64.

Kim, Y. S. and K. Kim (2006, October). How Important is the Intermediate Input Channel in Explaining Sectoral Employment Comovement over the Business Cycle? *Review of Economic Dynamics 9*(4), 659–682.

Kongsamut, P., S. Rebelo, and D. Xie (2001). Beyond balanced growth. *The Review of Economic Studies 68*(4), 869–882.

Krusell, P., L. E. Ohanian, J.-V. Rios-Rull, and G. L. Violante (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica 68*(5), 1029–1053.

Kydland, F. E. and E. C. Prescott (1982). Time to build and aggregate fluctuations. *Econometrica 50*(6), 1345–1370.

Lewis, W. A. (1954). Economic development with unlimited supplies of labour. *The Manchester School 22*(2), 139–191.

Long, John B, J. and C. I. Plosser (1983, 2). Real Business Cycles. *Journal of Political Economy 91*(1), 39–69.

Ngai, L. R. and C. A. Pissarides (2007, March). Structural Change in a Multisector Model of Growth. *American Economic Review 97*(1), 429–443.

Ravn, M. O. and H. Uhlig (2002). On adjusting the hodrick-prescott filter for the frequency of observations. *The Review of Economics and Statistics 84*(2), 371–375.

Smets, F. and R. Wouters (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *The American Economic Review 97*(3), 586–606.

Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters 20*(2), 177–181.

Uzawa, H. (1962). Production functions with constant elasticities of substitution. *Review of Economic Studies 29*(4), 291–299.