Learning Firm Conduct: Pass-Through as a Foundation for Instrument Relevance*

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Abstract

Researchers often test firm conduct models using pass-through regressions or instrumental variables (IV) methods. The former has limited applicability; the latter relies on potentially irrelevant instruments. We show the falsifiable restriction underlying the IV method generalizes the pass-through regression, and cost pass-through differences are the economic determinants of instrument relevance. We analyze standard instruments' relevance and link instrument selection to target counterfactuals. We illustrate our findings via simulations and an application to the Washington marijuana market. Testing conduct using targeted instruments, we find the optimal ad valorem tax closely matches the actual rate.

KEYWORDS: Testing conduct, pass-through, instrument relevance, falsification

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1 Introduction

Understanding how firms behave is a central objective in economics research. As firm conduct is frequently unknown, researchers have developed various methods to infer it from data. An influential idea in the early empirical industrial organization (IO) literature was to utilize the implications of cost pass-through rates to test firm conduct.[1] The simplest implementation in Sumner (1981) estimates pass-through from a regression of prices on unit taxes. Because different oligopoly models predict different pass-through levels (e.g., Weyl and Fabinger, 2013), the estimates from these regressions reject incorrect models of firm behavior. This approach has been applied across multiple fields of economics, including IO, trade, and public finance.[2]

As noted in the literature (e.g., Bulow and Pfleiderer, 1983; MacKay, Miller, Remer, and Sheu, 2014), the pass-through regression approach to infer conduct is only valid in a narrow class of models and when a special cost shifter (e.g., unit tax or wholesale price) is observed in the data. An alternative approach, originating with Bresnahan (1982) and Lau (1982), relies on exogenous variation in market conditions to distinguish models. In a differentiated products environment, Berry and Haile (2014) formalize this intuition into a falsifiable restriction implemented with a set of potentially exogeneous instruments. While their approach is widely applicable, it is crucial to assess the relevance of instruments for falsification, i.e., their ability to distinguish misspecified models from the truth.

In this paper, we develop a framework for understanding the economic determinants of instrument relevance when falsifying models of firm conduct. We find that differences in particular features of cost pass-through matrices across oligopoly models underpin falsifiability. This has several practical implications for learning conduct, of which we highlight three.

First, we show that the falsifiable restriction in Berry and Haile (2014) coincides with the regression approach in Sumner (1981) in the limited class of models with constant pass-through – the only class where the regression approach is valid (MacKay et al., 2014). Thus, the instrument-based approach generalizes the pass-through regression approach.

Second, we use our framework to evaluate the relevance of various conduct instruments. In general, we find that rival cost shifters, product characteristics, and tax rates have different empirical content; each targets different features of the model's pass-through matrix and other characteristics. We characterize exactly the aspects of conduct each instrument targets.

Third, we show how this characterization can guide instrument selection for particular counterfactual analyses. For example, computing a counterfactual Laffer curve to determine

[1] See, for example, Sumner (1981); Sullivan (1985); Panzar and Rosse (1987).

[2] For recent applications of this method see, e.g., Pless and Van Benthem (2019) and Brugués and De Simone (2024).

a revenue-maximizing tax rate depends on both demand and firm conduct (e.g., Miravete, Seim, and Thurk, 2018; Hollenbeck and Uetake, 2021). The features of conduct that determine the optimal tax rate are exactly those targeted by an instrument based on past variation in that tax rate. When such variation is not available, our framework illustrates the trade-offs between other possible instruments.

While our falsification results are presented in a population context, they directly inform finite sample testing procedures based on the Rivers and Vuong (2002) (RV) model selection test. Lack of falsification due to irrelevant instruments leads to degeneracy of the RV test and unreliable inference (Duarte, Magnolfi, Sølvsten, and Sullivan, 2024). Through Monte Carlo simulations, we illustrate the usefulness of our framework for testing models of conduct with RV, highlighting the importance of ex-ante instrument selection for both testing specific models and informing counterfactual outcomes. Our results also reveal the economic determinants of the ex-post weak instrument diagnostic in Duarte et al. (2024).

As an application of our results, we consider the problem of setting the optimal ad valorem tax rate in the Washington State marijuana market in 2016. Tax revenue is a key consideration for governments legalizing marijuana, and Hollenbeck and Uetake (2021) show the importance of accounting for firm market power to determine the optimal tax in this market. Following legalization, it is unclear to what extent retailers behave strategically. Escudero (2018) suggests that immediately following legalization (2014-2016), retailers may have used a simple rule of setting prices equal to twice marginal costs, a strategy referred to as Keystone pricing after a jewelers' trade publication in which it was widely promoted in the late 1800s. Hollenbeck, Hristakeva, and Uetake (2024) provide reduced-form evidence of a more nuanced relationship between markups and the competitive environment in later years (2018-2022).

Motivated by these findings, we use our framework to formally test the strategically sophisticated Bertrand-Nash pricing model against the simpler Keystone pricing model in a data sample from 2014-2017. These two models reflect two "extremes" of sophistication in how firms respond to competition, and it is *a priori* unclear which will better fit the data. We first estimate a demand system similar to Hollenbeck and Uetake (2021). We then test the Bertrand-Nash model against Keystone pricing using ad valorem tax instruments, which our framework shows are relevant for testing these models and will falsify models that yield incorrect optimal tax predictions. Our test indicates that Keystone provides a better fit than Bertrand-Nash during our sample period. Based on these findings, we solve for the optimal state-level ad valorem tax in 2016. The revenue maximizing state-level tax rate under Keystone is 38%, almost identical to the 37% rate the government adopted in 2015. Alternative conduct models imply different optimal taxes.

This paper complements existing work (Backus, Conlon, and Sinkinson, 2021; Duarte et al., 2024) that advocates a model selection approach and offers inferential advances in conduct testing. The falsification framework we develop can help clarify the economic determinants of instrument strength for the RV test and inform ex ante instrument selection. We illustrate this connection in our Monte Carlo simulations and our empirical application, which both implement testing with RV.

Our work is also related to recent work on demand curvature and pass-through. Weyl and Fabinger (2013) link pass-through to both firm conduct and demand curvature. Miravete, Seim, and Thurk (2024) show how standard discrete choice demand models can be made flexible, so as to produce a full range of demand curvature. Our results underscore the need for flexible demand specifications that can accurately capture pass-through patterns.

We also contribute to a long line of literature on optimal tax policy, in which the Laffer curve and other aspects of tax incidence have been studied extensively. Following the seminal contribution of Ramsey (1927), this literature has traditionally imposed the assumption of perfect competition (Auerbach, 1985, 2002; Slemrod, 1990). However, there is extensive theoretical evidence that conduct has important implications for tax policy. For example, Suits and Musgrave (1953) and Anderson, de Palma, and Kreider (2001) show that the relative efficiency of unit and ad valorem taxation depends on the form of market conduct. Recent work by Kroft, Laliberté, Leal-Vizcaíno, and Notowidigdo (2023) and Brugués and De Simone (2024) attempts to perform inference on conduct to better-inform tax policy. These papers use "conduct parameter" and "conjectural variations" approaches; like pass-through regressions, these approaches are only micro-founded for a narrow range of models (Makowski, 1987; Lindh, 1992; Corts, 1999). We contribute to this literature by providing guidance on which features of conduct – captured by the pass-through matrix – are most relevant to tax policy. Our insights can help guide instrument choice when prior variation in tax rates is unavailable. Relative to approaches based on pass-through regressions and conjectural variations, our approach is valid in a much broader range of settings, including those used in state-of-the-art structural equilibrium models.

The paper proceeds as follows. Section 2 describes the environment and shows that the exclusion restriction approach generalizes the pass-through approach. Section 3 presents our main results showing how differences in pass-through matrices form the economic determinants of falsifiability. Section 4 compares the relevance of demand side, cost side, and tax instruments. Section 5 motivates instrument selection for optimal tax counterfactuals. In Section 6, we perform Monte Carlo simulations to show the applicability of our results to a setting using the RV test. Section 7 develops an application to the Washington marijuana market. Section 8 concludes. Proofs are found in Appendix A.

2 Two Approaches to Falsification

We consider falsification of models of firm conduct using data across many markets.[3] A set of multi-product firms compete in each market t; for simplicity, we assume the same set of J products is sold in every market, although their characteristics may differ across markets. For each product and market combination (j,t), the researcher observes price p_{jt}, market share s_{jt}, a vector of product characteristics x_{jt}, and a vector of cost shifters w_{jt} that affects the product's marginal cost. For any variable a_{jt}, let a_t denote the vector of values for all products j in market t. We assume that, for all markets t, the demand system is $s_t = \mathcal{s}(p_t, x_t, \xi_t)$, where ξ_t is a vector of unobserved product characteristics. To focus on the supply side, we assume that the demand system is already known to the researcher. We normalize market size to 1, so that quantity q_{jt} and market share s_{jt} can be used interchangeably.

The data in each market t are generated by equilibrium play in some true model of firm behavior, characterized by a system of first-order conditions,

$$p_t = \Delta_{0t} + c_{0t}, \tag{1}$$

where Δ_{0t} is the true vector of markups in market t and c_{0t} is the true vector of marginal costs. Because true markups (or, equivalently, true costs) are unobserved, we consider theoretically plausible models to be falsified by the data. Under any imposed model m, we can calculate the implied markups Δ_{mt} as a known function of observables and demand primitives. Implied marginal costs c_{mt} can therefore also be calculated via a model-specific version of the first-order conditions in Equation (1), as $p_t - \Delta_{mt}$.

For the first-order conditions of any model m to characterize a well-defined empirical model, we require the following, analogous to Assumption 13 in Berry and Haile (2014):

Assumption 1. *(Equilibrium Uniqueness)* For any model m, including the true model, either there exists a unique equilibrium, or the equilibrium selection rule is such that the same p_t arises whenever the vector (c_{mt}, x_t, ξ_t) is the same.

Several methods have been proposed to learn whether the imposed model m is falsified by the data. We focus on two in the next subsections: the pass-through approach in Sumner (1981), and the exclusion restriction approach in Berry and Haile (2014).

[3] Falsification encompasses cases of nested and non-nested models. We focus on the latter. For the former, researchers can pursue identification of conduct; we discuss the usefulness of our framework for this approach in Appendix B.

2.1 Pass-through Regressions to Learn Conduct

Early tests of market power (e.g., Sumner, 1981) relied on a specific implication of different models of conduct: cost (or, equivalently, unit-tax) pass-through. In a simple class of "rule-of-thumb" pricing models, where firms set prices as a multiple of cost, own cost pass-through can be expressed as a constant scalar ϕ_m across products and markets. Marginal cost pricing ($\phi_m = 1$) and Keystone pricing ($\phi_m = 2$) are examples in this class. If data are generated by a model $m = 0$ in this class, Equation (1) becomes $p_{jt} = \phi_0 c_{0jt}$. Because cost pass-through is represented by a scalar parameter, it can be measured via linear regression. To implement the regression, we assume that marginal costs are constant in quantity, linear in observable cost shifters w_{jt}, and additively separable in an unobserved shock, or $c_{0jt} = \mathrm{w}_{jt}\gamma + \omega_{0jt}$. While the researcher can control for w_{jt}, she cannot keep ω_{0jt} fixed across markets.

With these assumptions, firms set prices $p_{jt} = \phi_0 \left(\mathrm{w}_{jt}\gamma + \omega_{0jt}\right)$. If we knew the value of γ, then the true own pass-through ϕ_0 could be consistently estimated by regressing p_{jt} on w_{jt}. While γ is unknown for a vector of generic cost shifters, there may be components of cost for which $\gamma = 1$. These include unit taxes which are levied on the firm (τ_{jt}) and wholesale prices (p^W_{jt}). As in Sumner (1981), suppose we observe a market-level unit tax τ_t for which there is variation across markets. We can residualize all variables, including τ_t with respect to the observed cost shifters w_{jt}, excluding τ_t. Then, we can estimate own cost pass-through by regressing p_{jt} on τ_t: $\phi_0 = \beta_{PTR} = \mathrm{cov}(\tau_t, p_{jt})/\mathrm{var}(\tau_t)$. Hence, when $\beta_{PTR} \neq \phi_m$, one can falsify the rule-of-thumb model for which the firm prices at ϕ_m times marginal cost. For example, if $\beta_{PTR} \neq 1$, marginal cost pricing is falsified.

However, the pass-through regression approach has a few conceptual and practical issues. First, the required variation in taxes or observed cost components may not be available in the data. Second, outside of constant pass-through environments, such as the one described above or the homogenous product environments in Bulow and Pfleiderer (1983), own and rival cost pass-through are not structural parameters. Instead, in market t, cost pass-through under model m is represented by the $J \times J$ matrix P_{mt}, which varies with realizations of both observable and unobservable determinants of demand and cost. Consequently, it is generally not possible to recover the elements of P_{mt} via regression (MacKay et al., 2014).

2.2 Exclusion Restrictions to Learn Conduct

Following the seminal work of Bresnahan (1982) and Lau (1982), an alternative approach to learn conduct in the presence of unobservable shocks to demand and cost is to leverage exogenous variation in market conditions. Berry and Haile (2014) generalized this intuition: if a researcher can construct instruments z_{jt} that are mean independent of the unobserved cost shocks under the true model, the exclusion restriction can serve as a falsifiable restriction.

Assumption 2. *(Instrument Exogeneity)* Marginal costs are $c_{0jt} = \bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt}) + \omega_{0jt}$ for each j, and z_{jt} is a vector of K excluded instruments such that $E[\omega_{0jt} \mid \mathrm{w}_{jt}, z_{jt}] = 0$ *a.s.*[4]

We further simplify the environment with the following assumption:

Assumption 3. *(Constant Marginal Cost)* Marginal costs are constant in quantities and only depend on the observable cost shifters w_{jt}, or $\bar{c}_{0j}(q_{jt}, \mathrm{w}_{jt}) = \bar{c}_{0j}(\mathrm{w}_{jt})$ for all j. Further, $\bar{c}_{0j}$ is differentiable in w_{jt}, with partial derivatives $\frac{\partial \bar{c}_{0j}}{\partial \mathrm{w}_{jt}} \neq 0$ everywhere.

For our running example and our empirical application, we will assume that marginal costs are linear in cost shifters, $\bar{c}_{0j}(\mathrm{w}_{jt}) = \mathrm{w}_{jt}\gamma$, as is standard in the empirical literature; but we make no such assumption for our general falsification results.

Berry and Haile (2014) point out that several sources of variation are typically exogenous, which can then be used to construct instruments. These include rival cost shifters and own and rival product characteristics. We refer throughout to *cost side instruments* and *demand side instruments*, respectively, as those formed with this variation.

When true costs and markups are unobserved, the exclusion restriction summarizes what we know about the true model: $E[\omega_{0jt} \mid \mathrm{w}_{jt}, z_{jt}] = 0$ holds for the true cost function. For a candidate model m and candidate cost function $\bar{c}_{mj}$, we can use Equation (1) to define ω_{mjt}, a model specific analog of ω_{0jt}, as $\omega_{mjt} = p_{jt} - \Delta_{mjt} - \bar{c}_{mj}(\mathrm{w}_{jt})$. Thus, $E[\omega_{mjt} \mid \mathrm{w}_{jt}, z_{jt}]$ provides a natural way to measure a model's fit. When $E[\omega_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] = 0$,[5] the model has the same fit as the true model, so it is not falsified by the instruments z_{jt}. Conversely, when there are no cost functions $\{\bar{c}_{mj}(\mathrm{w}_{jt})\}_{j=1}^{J}$ satisfying this falsifiable restriction almost surely over the values of w_{jt} and z_{jt}, the model is *falsified by the instruments* z_{jt}.

While instruments may be exogenous, they need not be relevant for falsification, meaning the falsifiable restriction may not be violated for a model m that is not the truth. However, determining instrument relevance from the falsifiable restriction is not obvious. To focus on the economic determinants of instrument relevance, we next connect the intuition of the pass-through regression to the instrument-based approach.

2.3 Connecting Pass-through Regression to Exclusion Restriction

Consider again the setting of Section 2.1: the true model is in the rule-of-thumb class, we have variation in unit taxes τ_t, and have residualized all variables with respect to w_{jt}. As we

[4]Here, as in the rest of the paper, the expectation is taken over realizations of unobservables, and *almost surely (a.s.)* refers to realizations of the exogenous observables $(\mathrm{w}_{jt}, z_{jt})$.

[5]This expression is the analogue of the condition in Theorem 9 of Berry and Haile (2014).

are making a connection to regression, we implement the falsifiable restriction as a covariance restriction using τ_t as an instrument. For models m in the rule-of-thumb class, where $p_{jt} = \phi_m c_{jt} = \phi_m \left(\bar{c}_{jt} + \tau_t + \omega_{jt}\right)$, we can rewrite $E[\omega_{mjt}\tau_t] = \phi_m^{-1} E[p_{jt}\tau_t] - E[(\tau_t + \omega_{0jt})\tau_t]$.[6] By Assumption 2, τ_t is uncorrelated with ω_0, so that model m is falsified when $\phi_m \neq (E[\tau_t\tau_t])^{-1} E[p_{jt}\tau_t]$. Since $(E[\tau_t\tau_t])^{-1} E[p_{jt}\tau_t] = \beta_{PTR}$, falsifying a rule-of-thumb model with unit tax instruments amounts to comparing ϕ_m to the coefficient of the pass-through regression. In this limited setting, the instrument-based approach compares measured pass-through with pass-through implied by model m and thus coincides with the regression approach.

This connection can be further used to understand instrument relevance. Consider again the example of falsifying a rule-of-thumb model, but now using a rival firm's cost shifter as an excluded instrument, which we denote z_{jt}. Model m is not falsified by this instrument, because both $E[p_{jt}z_{jt}] = E[\tau_t z_{jt}] = 0$. Economically, in the rule-of-thumb class, pass-through of a rival's cost is zero, so variation in a rival's cost shifters has no effect on market outcomes: prices and shares. Notice that this can be cast as an irrelevance problem for an IV version of the pass-through regression, where z_{jt} is used to instrument for τ_t. In the next section, we will show that differences in pass-through continue to underpin instrument relevance beyond this simple environment where pass-through is summarized by a parameter.

3 Role of Pass-Through for Falsification

To gain insight into the economic determinants of instrument relevance, it is useful to follow Backus et al. (2021) and rewrite the falsifiable restriction in terms of markups, an economic implication of the model. As prices in the data are generated from the true model in Equation (1), we can express the model specific unobservable ω_{mjt} as $\omega_{mjt} = \Delta_{0jt} - \Delta_{mjt} + \bar{c}_{0j}(\mathrm{w}_{jt}) - \bar{c}_{mj}(\mathrm{w}_{jt}) + \omega_{0jt}$. From this, we can restate the falsifiable restriction as follows.[7]

Lemma 1. *Under Assumptions 1-3, model m is falsified by instruments z_{jt} if and only if for some j there exists no function $\bar{c}_{mj}(\mathrm{w}_{jt})$ such that*

$$E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] = \bar{c}_{mj}(\mathrm{w}_{jt}) - \bar{c}_{0j}(\mathrm{w}_{jt}) \qquad a.s.$$

We want to understand the economic features of a model that underlie the empirical content of this statistical condition and therefore distinguish models of conduct.[8] Since this

[6] Note that residualized variables have zero mean, so for two variables x and y, $E[xy] = \mathrm{cov}(x, y)$.

[7] Proofs of all lemmas, propositions, and corollaries are in Appendix A.

[8] Previous work (Backus et al., 2021; Duarte et al., 2024) has explored statistical implications of this condition in the context of the RV model selection test.

condition involves the conditional expectation of a variable with respect to an instrument, a useful step is to consider how the variable moves with a change in the instrument. Thus, it is useful to restate Lemma 1 in terms of the marginal impacts of the instruments on markups. To do this, we will assume markups do not move discontinuously as instruments change:[9]

Assumption 4. *(Continuous Markups)* For any model m, for any j, $E[\Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}]$ is absolutely continuous in z_{jt}.

Based on Lemma 1, for a model not to be falsified, the conditional expectation of $\Delta_{0jt} - \Delta_{mjt}$ must match the difference in implied cost at each value of the instruments z_{jt}. As instruments are excluded from cost, a marginal change in any of the K instruments has no effect on the implied costs for either model. This means that if a model is not falsified, the impact of the instruments on the conditional expectation of $\Delta_{0jt} - \Delta_{mjt}$ must be zero. We focus on the limit of this difference,

$$\lim_{h\to 0} \frac{E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt} = \tilde{z}_{jt} + h_k] - E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt} = \tilde{z}_{jt}]}{h}$$

where h_k is a K-vector of zeros with the scalar h in the k-th position. The marginal effect of the k-th instrument $z_{jt}^{(k)}$ on the conditional expectation of $\Delta_{0jt} - \Delta_{mjt}$ is the average difference in the marginal effect of $z_{jt}^{(k)}$ on Δ_{0jt} and Δ_{mjt}, giving the following:

Lemma 2. *Under Assumptions 1-4, model m is falsified by instruments z_{jt} if and only if for some j and k,*

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p.$$

We illustrate the intuition behind Lemma 2 in the following example:

Example 1: In this and every subsequent example, we consider a simple environment with two single-product firms and logit demand, so that market shares $j \in \{1, 2\}$ are given by

$$s_{jt} \quad = \quad \frac{\exp(x_{jt}\beta - \alpha p_{jt})}{1 + \exp(x_{1t}\beta - \alpha p_{1t}) + \exp(x_{2t}\beta - \alpha p_{2t})},$$

where x_{jt} are characteristics of product j in market t and α and β are coefficients. Further suppose that there is no unobservable variation, so that demand (as a function of price) and the cost of firm 1, c_{01t}, remain fixed.

[9] Assumption 4 holds for all models in this paper, and for all standard models.

First, suppose that the true model generating the data is Bertrand-Nash competition in prices, which we will hereafter refer to as Bertrand, and we want to use rival cost shifters to falsify the model of Keystone pricing, under which $p_{jt} = 2c_{mjt}$ and $\Delta_{mjt} = c_{mjt}$. Suppose that we observe variation in the cost shifter of firm 2, w_{2t}, so that $z_{1t} = \mathrm{w}_{2t}$. Under Lemma 2, Keystone is falsified if the instruments differentially move the Keystone and Bertrand markups for firm 1. Under the true Bertrand model, when w_{2t} increases, both firms' equilibrium prices increase, and since c_{01t} did not change, Δ_{01t} increases by the same amount as p_{1t}.[10] Under the Keystone model, however, firm 1's equilibrium price does not change with variation in w_{2t}, all else equal. To rationalize the increase in p_{1t}, the Keystone model would impute an increase in firm 1's marginal cost c_{m1t} and markups Δ_{m1t} by *half* the amount of the change in p_{1t}. The difference between $\frac{\mathrm{d}\Delta_{01t}}{\mathrm{dw}_{2t}}$ and $\frac{\mathrm{d}\Delta_{m1t}}{\mathrm{dw}_{2t}}$ is what allows us to falsify Keystone.$\bullet$

The derivative of markups with respect to the instruments is an object whose properties depend on the economics of the firm conduct model, so this lemma allows us to connect the econometric perspective on falsifiability with a more theoretical view. To falsify a model, variation in the instruments must induce differential changes in the implied markups for that model and the true model. However, since demand and cost shocks are unobserved, it is not possible to isolate changes solely caused by the instruments; we therefore need the marginal impact of the instruments to differ across the two models when we average over the unobserved shocks.

Even beyond the restricted settings considered in Section 2.1 where the true own cost pass-through can be consistently estimated via regression, it is still largely the cost pass-through of a model that determines falsifiability. To see this, note that the markups for the true data generating process and the model to be falsified are functions of two endogenous variables, market shares and prices. As the demand system makes market shares a function of prices, we can write the vector Δ_{mt} as a function of prices, instruments and other exogenous variables, $\Delta_{mt} = \Delta_m(p_t, z_t, \mathrm{w}_t, x_t, \omega_{mt}, \xi_t)$.[11]

Instruments may affect markups either directly, or through prices. In light of Lemma 2, we are interested in the average difference between the causal effects of an instrument $z_{jt}^{(k)}$

[10] In Example 2, we will derive the cost pass-through matrix for the Bertrand model, and see that it has positive off-diagonal terms. The economic intuition is that when firm 2 raises its price in response to an increase in costs, this decreases the own-price elasticity of firm 1, leading firm 1 to raise its price as well.

[11] This holds even if markups depend directly on marginal costs as in Keystone pricing, since marginal costs are themselves determined by w_{jt} and ω_{mjt}.

on model m and the truth. Letting $(A)_j$ denote the j^{th} row of a vector or matrix, we have

$$\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} = \underbrace{\frac{\partial\Delta_{0jt}}{\partial z_{jt}^{(k)}} - \frac{\partial\Delta_{mjt}}{\partial z_{jt}^{(k)}}}_{\text{Direct Effect}} + \underbrace{\left(\frac{\partial\Delta_{0t}}{\partial p_t} - \frac{\partial\Delta_{mt}}{\partial p_t}\right)_j \frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}}}_{\text{Indirect Effect}}$$

where $p_0(\cdot)$ is the function mapping primitives to equilibrium prices under the true model. From this expression we see two distinct effects of instruments on the difference in markups. The first is the *direct effect* of instruments, or $\frac{\partial\Delta_{0jt}}{\partial z_{jt}^{(k)}} - \frac{\partial\Delta_{mjt}}{\partial z_{jt}^{(k)}}$. This element is non-zero whenever instruments such as product characteristics differentially enter the markup functions of model m and the truth. The second term represents the *indirect effect*, which happens through prices. We further investigate the economic content of this term. For expositional clarity, we make a simplifying assumption:

Assumption 5. *(No Direct Effect of Costs)* The markup functions Δ_{0t} and Δ_{mt} for both the true model and the model to be falsified do not depend directly on marginal costs.

Most standard conduct models based on profit maximization satisfy this assumption. Moreover, as long as the mapping from marginal costs to prices is invertible, any dependence of markups on costs can be restated as a dependence on prices to satisfy Assumption 5. For example, under Keystone pricing $\Delta_{Kjt} = c_{jt}$, but we can alternatively write $\Delta_{Kjt} = \frac{1}{2}p_{jt}$ to remove the direct dependence on costs and satisfy Assumption 5, which therefore serves more as a labeling convention than a substantive assumption.[12]

For any model m we can compute the cost pass-through matrix P_{mt} via the Implicit Function Theorem. Writing the first-order conditions for any model m in market t as

$$F_m(p_t, c_t) = p_t - c_t - \Delta_{mt} = 0$$

equilibrium prices under model m are an implicit function of costs, $p_t = p_m(c_{mt})$, defined as the solution to $F_m(p_m(c_{mt}), c_{mt}) = 0$. Thus, for any model m, p_m is defined implicitly by these first-order conditions; under Assumption 5, the Implicit Function Theorem gives

$$P_{mt} = \frac{\mathrm{d}p_m(\cdot)}{\mathrm{d}c_t} = -\left[\frac{\partial F_m}{\partial p_t}\right]^{-1}\frac{\partial F_m}{\partial c_t} = (I - H_{\Delta_{mt}})^{-1}$$

where $H_{\Delta_{mt}} = \frac{\partial\Delta_{mt}}{\partial p_t}$. In the indirect effect, we can rewrite the difference in price derivatives of markups in terms of inverse pass-through matrices, because $P_{mt}^{-1} = \left(I - \frac{\partial\Delta_{mt}}{\partial p_t}\right)$ as long as

[12] Assumption 5 can fail when the mapping from costs to prices is not invertible – for example, if a firm priced a product at \$50 whenever marginal costs were between \$30 and \$40.

P_{mt} is invertible. We make the assumption that it is:[13]

Assumption 6. *(Invertibility of Pass-throughs)* For any model m, in any market t, the pass-through matrix P_{mt} has full rank.

Example 2: We return to the "example environment" of two single-product firms under simple logit demand, and calculate equilibrium markups and pass-through matrices for four models we will use extensively for intuition. First is marginal cost pricing, $p_{jt} = c_{0jt}$, under which markups are $\Delta_{MCjt} = 0$, and pass-through and inverse pass-through matrices are $P_{MCjt} = P^{-1}_{MCjt} = I$. Second is the Keystone model, where prices are set at twice marginal costs; this gives markups $\Delta_{Kt} = \frac{1}{2}p_t$, cost pass-through matrix $P_{Kt} = 2I$, and inverse pass-through matrix $P^{-1}_{Kt} = \frac{1}{2}I$. Next is Bertrand price competition, which in this setting gives markups, inverse pass-through, and pass-through matrices

$$\Delta_{Bt} = \begin{bmatrix} \frac{1}{\alpha(1-s_{1t})} \\ \frac{1}{\alpha(1-s_{2t})} \end{bmatrix}, \quad P^{-1}_{Bt} = \begin{bmatrix} \frac{1}{1-s_{1t}} & -\frac{s_{1t}s_{2t}}{(1-s_{1t})^2} \\ -\frac{s_{1t}s_{2t}}{(1-s_{2t})^2} & \frac{1}{1-s_{2t}} \end{bmatrix}, \quad \text{and} \quad P_{Bt} = \kappa_{Bt} \begin{bmatrix} \frac{1}{1-s_{2t}} & \frac{s_{1t}s_{2t}}{(1-s_{1t})^2} \\ \frac{s_{1t}s_{2t}}{(1-s_{2t})^2} & \frac{1}{1-s_{1t}} \end{bmatrix}$$

where $s_{0t} = 1 - s_{1t} - s_{2t}$ and $\kappa_{Bt} = s^{-1}_{0t}(1-s_{1t})^2(1-s_{2t})^2$. Finally, there is the differentiated-products version of Cournot competition in quantities, where each firm chooses a market share s_{jt} (taking the other firm's share as given) to maximize profits. This gives markups, inverse pass-through, and pass-through

$$\Delta_{Ct} = \begin{bmatrix} \frac{1-s_{2t}}{\alpha s_{0t}} \\ \frac{1-s_{1t}}{\alpha s_{0t}} \end{bmatrix}, \quad P^{-1}_{Ct} = \begin{bmatrix} \frac{1-s_{2t}}{s_{0t}} & 0 \\ 0 & \frac{1-s_{1t}}{s_{0t}} \end{bmatrix}, \quad \text{and} \quad P_{Ct} = \begin{bmatrix} \frac{s_{0t}}{1-s_{2t}} & 0 \\ 0 & \frac{s_{0t}}{1-s_{1t}} \end{bmatrix}$$

Assumption 6 holds for all four conduct models. Also in this simple environment, the Cournot model has zero pass-through of rival's costs (or zero off-diagonal elements for P_{Ct}) like marginal cost pricing and Keystone pricing. This will be important in future examples. •

Breaking the terms in Lemma 2 into the direct and indirect effects yields the following:

[13] This assumption has economic content, as it requires that each product has non-zero pass-through for at least one cost (either own or rival) in the market. Moreover, pass-through vectors for each product cannot be linear combinations of those of other products, meaning that costs must affect different products in a distinct way. This is satisfied in most models, as the pass-through of own cost that is measured by the main diagonal of P_{mt} is typically different than the pass-through of rival costs.

Proposition 1. *Under Assumptions 1-6, a model m is falsified by instruments z_{jt} if and only if for some j and k,*

$$E\left[\frac{\partial \Delta_{0jt}}{\partial z_{jt}^{(k)}} - \frac{\partial \Delta_{mjt}}{\partial z_{jt}^{(k)}} + \left(P_{mt}^{-1} - P_{0t}^{-1}\right)_j \frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p. \tag{2}$$

This proposition casts falsification in terms of the direct and indirect effect of the instruments. When the instruments move the markup functions directly and differentially, this will likely enable falsification. Furthermore, if instruments indirectly affect markups through prices, falsification depends on $(P_{mt}^{-1} - P_{0t}^{-1})$, the difference in inverse pass-throughs of the true model and model m. Similar to the intuition obtained from the pass-through regression, differences in inverse pass-throughs (and therefore pass-throughs) permit falsification, though only if instruments move prices under the true model, i.e. $\frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} \neq 0$.

The general framework we develop has practical applications. We highlight two in the next sections: evaluating the relevance of specific instruments, and motivating instrument selection when a researcher's objective is to perform a particular counterfactual.

4 Evaluating Instrument Relevance

Next, we evaluate the relevance of instruments commonly used in the literature. We start with standard cost side instruments (based on rival cost shifters) and demand side instruments (based on product characteristics). We then augment Proposition 1 for settings with taxes, and show the relevance of tax instruments.

4.1 Cost Side Instruments

The implications of Proposition 1 are particularly stark when the instruments are formed with rival cost shifters. Under Assumption 5, the instruments have no direct effect on markups, and enter either markup function only through $p_0(\cdot)$. Note that since the marginal cost function $\bar{c}_{mj}$ is not observed directly, a firm's own cost shifters are irrelevant instruments for conduct: for any model m, we can always specify a cost function $\bar{c}_{mj}$ giving $E[\omega_{mjt}|\mathrm{w}_{jt}] = 0$.[14] Thus, only observable shifters of a *rival's* cost are plausibly relevant instruments.

Example 3: We return to the example environment. Suppose we want to falsify Bertrand competition when the true model is Cournot competition, using variation in the cost shifter of firm 2, w_{2t}, so that $z_{1t} = \mathrm{w}_{2t}$. Assume w_{2t} is scalar, and $\bar{c}_{02}(\mathrm{w}_{2t}) = \gamma \mathrm{w}_{2t}$. Under the true

[14] For any model m, define $\bar{c}_{mj}(\mathrm{w}_{jt}) = E[p_{jt}|\mathrm{w}_{jt}] - E[\Delta_{mjt}|\mathrm{w}_{jt}]$, and $E[\omega_{mjt}|\mathrm{w}_{jt}] = 0$ follows.

model, variation in w_{2t} only moves the price of firm 2, with $\frac{\mathrm{d}p_0}{\mathrm{d}z_{1t}} = \begin{bmatrix} 0 \\ \gamma\frac{s_{0t}}{1-s_{1t}} \end{bmatrix}$. In this simplified environment where there is no direct effect of the instruments under either model and where the unobservables are held fixed, falsification under Proposition 1 obtains if in any market t, $\left(P_{Bt}^{-1} - P_{Ct}^{-1}\right)_1 \begin{bmatrix} 0 \\ \gamma\frac{s_{0t}}{1-s_{1t}} \end{bmatrix} \neq 0$ or $\gamma\frac{s_{0t}}{1-s_{1t}}\left(\left(P_{Bt}^{-1}\right)_{12} - \left(P_{Ct}^{-1}\right)_{12}\right) \neq 0$. Thus, Bertrand is falsified by variation in w_{2t}, as the (1,2)-elements of P_{Bt}^{-1} and P_{Ct}^{-1} differ. If instead we wished to falsify either the Keystone model or marginal cost pricing, we would fail: pass-through matrices under both Cournot and the model to be falsified are diagonal, so $\left(P_{Ct}^{-1}\right)_{12} = \left(P_{Kt}^{-1}\right)_{12} = \left(P_{MCt}^{-1}\right)_{12} = 0$, and rival cost shifters are irrelevant for falsifying either of the two models. •

To move from Example 3 to our general environment, define the $J \times J$ matrix $[P_m^{-1}P_0]^\star$ by

$$[P_m^{-1}P_0]_j^\star \quad = \quad E\left[P_{mt}^{-1}P_{0t} \mid \mathrm{w}_{jt}, z_{jt}\right]_j.$$

That is, $[P_m^{-1}P_0]^\star$ is the expected value of $P_{mt}^{-1}P_{0t}$, but where the expectation in the j-th row is taken conditional on the realized values of product j's cost shifters and instruments $(\mathrm{w}_{jt}, z_{jt})$. Thus, $[P_m^{-1}P_0]^\star$ is a function of the full vector of observables (w_t, z_t), with each row depending on a different subset of those observables. This matrix arises from Equation (2) because $\frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}}$ will often be proportional to P_{0t}, so $(P_{mt}^{-1} - P_{0t}^{-1})\frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} \propto (P_{mt}^{-1}P_{0t} - I)$.

Given observable and unobservable variation, falsifiability depends on the matrix $[P_m^{-1}P_0]^\star$:

Corollary 1. *Suppose that for each product j, the vector of instruments z_{jt} includes a cost shifter of every rival product. Under Assumptions 1-6, model m is falsified by the instruments if with positive probability over (w_t, z_t), the matrix $[P_m^{-1}P_0]^\star$ is not diagonal.*

For intuition, put aside the expectation over unobservables, and focus on model m being falsified by rival cost shifters if $P_m^{-1}P_0$ is not diagonal. This is when $P_0 \neq P_m D$ for any diagonal matrix D, i.e., when some column of P_m is not a scalar multiple of the corresponding column of P_0. Assuming own cost pass-through rates are nonzero, this in turn happens if and only if there is some (j, ℓ) such that the ratio

$$\frac{\mathrm{d}p_{\ell t}}{\mathrm{d}c_{jt}} \bigg/ \frac{\mathrm{d}p_{jt}}{\mathrm{d}c_{jt}}$$

differs between model m and the true model. If instead this ratio is the same across the two models, then model m can be rationalized by the unknown $\frac{\partial \bar{c}_{mj}}{\partial \mathrm{w}_{jt}}$, leading to lack of falsification with cost side instruments.[15] For example, if P_{mt} and P_{0t} are always both diagonal matrices, then this ratio is always zero and model m is not falsified by cost side instruments. Thus,

[15]Suppose a shifter of cost j moves, resulting in changes to prices p_j and (potentially) p_ℓ. The change in p_j is proportional to $\frac{\mathrm{d}p_{0jt}}{\mathrm{d}c_{jt}}\frac{\partial \bar{c}_{0j}}{\partial \mathrm{w}_{jt}}$; the change in p_ℓ is proportional to $\frac{\mathrm{d}p_{0\ell t}}{\mathrm{d}c_{jt}}\frac{\partial \bar{c}_{0j}}{\partial \mathrm{w}_{jt}}$. The effect of the cost shifter

we can think of cost side instruments targeting the ratio of a rival product's pass-through of a product j's cost to product j's pass-through of its own cost.[16]

4.2 Demand Side Instruments

The General Case: In the general case, Proposition 1 characterizes when a model can be falsified using product characteristics. Two key differences arise between the case of cost side instruments just discussed and the case of demand side instruments. The first is that product characteristics are sometimes excluded from cost, allowing the researcher to use variation in both own and rival products' characteristics as valid instruments. This feature makes the indirect effect of instruments more useful: insofar as product characteristics are excluded from marginal cost, the ability to use own product characteristics as instruments will help the researcher trace out the full pass-through matrix, as opposed to relying on only the ratio of off-diagonal to diagonal elements for falsification. Thus, models with different diagonal pass-through matrices could still be falsified with demand side instruments. Second, in most standard conduct models, product characteristics will directly enter the markup function and therefore have a direct effect on the implied costs, so that $\frac{\partial \Delta_{mjt}}{\partial z_{jt}^{(k)}} \neq 0$. Adding an extra channel in general makes falsification easier, although there may be cases in which direct effects cancel out indirect effects, thus hindering falsification.

An Interesting Special Case: For a subset of conduct models and demand systems, we can simplify Equation (2) to a condition that is easier to interpret. For this result, we focus on models that satisfy two conditions. First, the first-order conditions (1) must be a system of equations containing only markups, market shares, and the derivatives of market shares with respect to prices; all other variables only enter into the first-order conditions through these. Second, demand must have a linear index structure. We state these conditions as formal assumptions next, beginning with the former.

Assumption 7. *(No Direct Effects of Primitives)* For both the true model and the model to be falsified, the first-order condition can be written as a function of only markups, market shares, and price derivatives of demand $\frac{\partial s_t}{\partial p_t}$.

Under Assumption 7, $\Delta_{mt} = \Delta_m\left(s_t, \frac{\partial s_t}{\partial p_t}\right)$ so that the implied markup function for any model m only depends directly on s_t and $\frac{\partial s_t}{\partial p_t}$. In Appendix C, we demonstrate a range

$\frac{\partial \bar{c}_{0j}}{\partial \mathrm{w}_{jt}}$ is not known ex ante; but for a model where $\frac{\mathrm{d}p_{m\ell t}}{\mathrm{d}c_{jt}} \Big/ \frac{\mathrm{d}p_{mjt}}{\mathrm{d}c_{jt}}$ differs from the truth, no value of $\frac{\partial \bar{c}_{mj}}{\partial \mathrm{w}_{jt}}$ can rationalize both price changes, so ω_{mjt} can't be mean-independent of w_{jt}.

[16] Falsifiability does not require every product's instruments z_{jt} to include a shifter of every rival product's cost, just that there be some pair (j, ℓ), $\ell \neq j$, where $[P_m^{-1} P_0]^\star_{j,\ell} \neq 0$ and z_{jt} includes a shifter of $c_{0\ell}$.

of conduct models which satisfy this assumption. These include all models where firms act to maximize their profits under some assumption about how rival firms will respond to their choices, such as Bertrand, Cournot, and Stackelberg; they also includes many collusive models, and models of altruistic firms who maximize a weighted sum of own profits and consumer surplus. For example, under Bertrand competition the implied markups are

$$\Delta_{Bt} \quad = \quad -\left[\Omega \odot \left[\frac{\partial s_t}{\partial p_t}\right]'\right]^{-1} s_t,$$

where Ω is the ownership matrix, with $\Omega_{ij} = 1$ if products i and j are sold by the same firm and zero otherwise. Here, we see that all primitives enter the equilibrium markup only through their effects on $\left(s_t, \frac{\partial s_t}{\partial p_t}\right)$. The assumption is also satisfied by marginal cost pricing, where $\Delta_{MCt} = 0$. However, the assumption fails in Keystone pricing, where p_t enters the implied markups directly: $\Delta_{Kt} = \frac{1}{2}p_t$. The assumption also fails in models where firms' objective functions depend in part on revenue rather than profit.

Under an additional assumption on the demand system, the effect of product characteristics becomes very similar to the effect of cost shifters on markups:

Assumption 8. *(Demand Index)* Demand depends on x_t and p_t only through a one-dimensional index $\delta_t = x_t\beta - \alpha p_t + \xi_t$, with $\alpha > 0$, all elements of β nonzero, and $s_t = \mathscr{s}(x_t, p_t, \xi_t, \cdot) = \mathscr{s}(\delta_t, \cdot)$.

This assumption is satisfied when the demand system is logit or nested logit; it is not generally satisfied for mixed logit demand models with random coefficients on either price or characteristics, but we expect the added variation in richer models to make falsification easier, not harder.[17]

Together, Assumptions 7 and 8 imply that the markup functions Δ_0 and Δ_m depend on prices and product characteristics only through δ_t. This, in turn, implies that x_t and c_t affect equilibrium markups only through the term $x_t\beta - \alpha c_t$, and therefore that the effects of marginal costs and product characteristics on markups are virtually identical. For intuition, consider the Bertrand model with single-product firms. Instead of choosing prices, we can think of firms directly choosing markups Δ_{jt} to maximize $(p_{jt} - c_{jt})\mathscr{s}_j(\delta_{jt}, \delta_{-jt}) = \Delta_{jt}\mathscr{s}_j\left(x_{jt}\beta - \alpha c_{jt} + \xi_{jt} - \alpha\Delta_{jt},\, \delta_{-jt}\right)$. The optimal Δ_{jt} depends on market primitives only through the term $x_{jt}\beta - \alpha c_{jt} + \xi_{jt}$. The same logic extends easily to all the models discussed in Appendix C.

[17] Alternatively, we can define $\delta_t = x_t\beta - p_t$ and $s_t = \mathscr{s}(x_t, p_t, \xi_t, \cdot) = \mathscr{s}(\delta_t, \xi_t, \cdot)$. The proceeding results still apply, and this admits mixed logit models as long as some product characteristic is a perfect substitute for income.

Example 4: In Example 3, cost side instruments could not falsify Keystone or marginal cost pricing when the truth is Cournot. Here, we consider falsification with demand side instruments. Let x_{jt} be a scalar characteristic of product j, and suppose $z_{1t} = x_{1t}$.

First, consider falsifying marginal cost pricing. Our example environment satisfies Assumption 8, and both the Cournot model and marginal cost pricing satisfy Assumption 7; so for both models, x_{1t} and c_{1t} affect firm 1's markups only through the term $x_{1t}\beta - \alpha c_{1t}$, and therefore $\frac{\mathrm{d}\Delta_{1t}}{\mathrm{d}x_{1t}} = -\frac{\beta}{\alpha}\frac{\mathrm{d}\Delta_{1t}}{\mathrm{d}c_{1t}}$. Under Lemma 2, then, the model of marginal cost pricing will be falsified if and only if $\frac{\mathrm{d}\Delta_{MC1t}}{\mathrm{d}c_{1t}} \neq \frac{\mathrm{d}\Delta_{C1t}}{\mathrm{d}c_{1t}}$. Indeed, under marginal cost pricing, markups are zero, and therefore $\frac{\mathrm{d}\Delta_{MC1t}}{\mathrm{d}c_{1t}} = 0$; while under Cournot, $\frac{\mathrm{d}\Delta_{C1t}}{\mathrm{d}c_{1t}} = [P_{Ct}]_{11} - 1 = -\frac{s_1}{s_0+s_1} \neq 0$; so the marginal cost pricing model is falsified.

Falsifying Keystone pricing is subtler, because markups $\Delta_{Kjt} = 0.5p_{jt}$ violate Assumption 7. However, since product characteristics do not move prices, demand side instruments have neither a direct nor an indirect effect on markups, and $\frac{\mathrm{d}\Delta_{K1t}}{\mathrm{d}x_{1t}} = 0$. As noted above, under Cournot, $\frac{\mathrm{d}\Delta_{C1t}}{\mathrm{d}x_{1t}} = -\frac{\beta}{\alpha}\frac{\mathrm{d}\Delta_{C1t}}{\mathrm{d}c_{1t}} = \frac{\beta}{\alpha}\frac{s_1}{s_0+s_1} \neq 0$, so the Keystone model is falsified as well.

Finally, if the truth were Keystone, demand side instruments would be irrelevant for falsifying marginal cost pricing, and vice versa, since $\frac{\mathrm{d}\Delta_{m1t}}{\mathrm{d}x_{1t}} = 0$ for both models. •

Under Assumptions 7 and 8, demand side instruments shift markups in the same way as marginal costs, suggesting that instrument relevance will be similar to cost side instruments. However, while own cost shifters are not excluded from costs (and therefore can't be used to show a violation of the exclusion restriction), own product characteristics *are* excluded. Whereas falsification using cost shifters depends on the *off-diagonal* terms of $P_m^{-1}P_0$, falsification using demand side instruments can use the diagonal terms as well. We can formalize this as follows:

Corollary 2. *Suppose that for every product j, the vector of instruments z_{jt} includes a product characteristic of every product. Under Assumptions 1-8, model m is falsified by the instruments z_t if with positive probability over (w_t, x_t), the matrix $[P_m^{-1}P_0]^\star$ is not equal to the identity matrix.*

The intuition is that the marginal effect of a product characteristic $x_{\ell t}^{(k)}$ on the inferred costs of product j under model m is proportional to $(I - P_{mt}^{-1}P_{0t})_{j\ell}$. Thus, if this term is nonzero in expectation for some values of observables, a change in $x_{\ell t}^{(k)}$ would lead to changes in the mean of c_{mjt}; since product characteristics used as instruments are excluded from costs, this would again violate the restriction that ω_{mjt} is mean-independent of the instruments.

Loosely, while falsifying a model with cost side instruments requires that $P_{mt}^{-1}P_{0t}$ is not diagonal on average, falsifying a model with demand side instruments only requires that $P_{mt}^{-1}P_{0t} \neq I$. Thus, when α and β are both non-zero, it is easier to falsify model m using product characteristics: under Assumptions 1-8 any model which is falsified by rival cost shifters is also falsified by product characteristics. This is because under Assumptions 7 and 8, the effect of product characteristics on markups is nearly identical to the effect of marginal costs, but scaled by demand system parameters β and α which are already known rather than through a cost function $\bar{c}_{mj}(\cdot)$ which is ex ante unknown. This allows demand side instruments to target the entire pass-through matrix, not just the ratio of off-diagonal to diagonal elements, allowing falsification whenever (loosely) $P_m \neq P_0$. In the more general case where either Assumption 7 or 8 does not hold, however, demand side instruments target a mix of pass-through and the direct effect of the instrument; this added effect likely helps in falsifying incorrect models, but leaves the source of this falsification more opaque.

4.3 Tax Instruments

Next, we consider tax rates as another source of exogenous variation. Suppose that governments levy market-level unit and/or ad valorem taxes, at rates τ_t and v_t, respectively.[18] For ad valorem taxes, we will work with ν_t, the fraction of the consumer's payment received by the firm, which is $1 - v_t$ when the tax is levied on the firm and $1/(1 + v_t)$ when it is levied on the consumer. For unit taxes, we will assume they are levied on firms.

In the presence of taxes, the after-tax version of the first order condition (1) is

$$\nu_t p_{jt} - \tau_t \quad = \quad \nu_t \Delta_{0jt} + c_{0jt}, \tag{3}$$

where $\nu_t p_{jt} - \tau_t$ is the after-tax revenue received by the firm for product j in market t.

Similar to Assumption 5, we assume that taxes do not directly enter markups:

Assumption 9. *(No Abnormal Effects of Tax Rates)* The markup functions Δ_{0t} and Δ_{mt} for the true model and the model to be falsified do not depend directly on tax rates.

This can be thought of as an assumption that taxes are fully salient and there is no avoidance/evasion. For example, under the Keystone model markups are $\Delta_{Kjt} = \frac{1}{2}p_{jt}$. In the presence of a unit tax and no ad valorem tax, this means $p_{Kjt} = 2(\tau_t + c_{Kjt})$; the firm perceives the unit tax as part of its marginal costs. With an ad valorem tax instead, $p_{Kjt} = 2c_{Kjt}/\nu_t$; the firm sets price such that after-tax revenue is twice marginal cost.

If variation in taxes is available, and assumed to be exogenous, we can use it to construct *tax instruments*. We can rewrite ω_{mt} as $\omega_{mjt} = \nu_t \Delta_{0jt} - \nu_t \Delta_{mjt} + \bar{c}_{0j}(\mathrm{w}_{jt}) - \bar{c}_{mj}(\mathrm{w}_{jt}) + \omega_{0jt}$. Falsification depends on the impossibility of finding a cost function $\bar{c}_{mj}$ giving $E[\omega_{mjt}|\mathrm{w}_{jt}, z_t] = 0$; since $E[\omega_{0jt}|z_t] = 0$ and the instruments are excluded from $\bar{c}_{0j}$ and $\bar{c}_{mj}$,

[18] Note that taxes apply uniformly to all products, so τ_t and ν_t are scalars, not vectors.

falsification occurs if $E[\nu_t(\Delta_{0jt} - \Delta_{mjt})|\mathrm{w}_{jt}, z_t]$ varies with the instrument. Under Assumption 9, $\frac{\partial \Delta_{mjt}}{\partial z_t} = \frac{\partial \Delta_{0jt}}{\partial z_t} = 0$ for either type of tax instrument; this lets us restate the falsifiable restriction as follows.

Proposition 2. *Under Assumptions 1-6 and 9, a model m is falsified by the tax instrument z_t if and only if for some j,*

$$E\left[\nu_t \left(P_{mt}^{-1} - P_{0t}^{-1}\right)_j \frac{\mathrm{d}p_0}{\mathrm{d}z_t} + \left(\Delta_{0jt} - \Delta_{mjt}\right) \frac{\mathrm{d}\nu_t}{\mathrm{d}z_t} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p. \tag{4}$$

That is, after ruling out any direct effect of the instrument entering the markup function, the marginal impact of an instrument on the across-model difference in (after-tax) markups still has two potential effects: the indirect effect through prices, and a new effect if the instrument is correlated with the ad valorem tax rate. We now discuss the relevance of unit taxes and ad valorem taxes in turn.

Unit Tax Instruments: We first consider unit tax instruments. For each model m, there is an implied "composite" marginal cost $\tilde{c}_{mjt} = \bar{c}_{mt}(\mathrm{w}_{jt}) + \tau_t + \omega_{mjt}$, such that $\nu_t(p_{jt} - \Delta_{mjt}) = \tilde{c}_{mjt}$. Therefore, marginal cost is composed of two types of observed cost shifters: w_{jt} which enters in an ex ante unknown way, and τ_t which enters additively with a known coefficient of 1. Since this coefficient is known ex ante, variation in a firm's *own* marginal cost induced by the unit tax can serve as exogenous variation for distinguishing conduct.

We noted in Example 4 that in our example environment, traditional demand side instruments fail to falsify a model of marginal cost pricing when the true model is Keystone pricing. Further, since marginal cost pricing and Keystone both have diagonal pass-through matrices, instruments based on rival cost shifters would also fail to falsify marginal cost pricing under Keystone truth. Here, we illustrate that unit tax instruments do allow falsification.

Example 5: Fix $\nu_t = 1$ (no ad valorem tax), and suppose that we observe variation in a unit tax which is levied on both firms, τ_t, so that $z_t = \tau_t$. Further suppose we are in a simplified environment where the unobservables and other cost shifters are held fixed. Under Keystone, $\Delta_{0jt} = \frac{1}{2}p_{jt}$, and under marginal cost pricing, $\Delta_{mjt} = 0$, so the tax rate τ_t does not enter directly into either markup function. Further, $P_{0t} = 2I$ and $P_{mt} = I$, and under the true model, $\frac{\mathrm{d}p_0}{\mathrm{d}\tau_t} = \left[\begin{smallmatrix}2\\2\end{smallmatrix}\right]$, since unit variation in τ_t moves both firms' costs by 1, increasing both prices by 2 under the true model (Keystone). Under Proposition 2, then, falsification occurs because $\left(P_{mt}^{-1} - P_{0t}^{-1}\right)_1 \frac{\mathrm{d}p_0}{\mathrm{d}\tau_t} = \left([1\ \ 0] - \left[\tfrac{1}{2}\ \ 0\right]\right)\left[\begin{smallmatrix}2\\2\end{smallmatrix}\right] \neq 0.$ •

In our general environment, we can establish the following:

Corollary 3. *Suppose that for each product j, the instrument z_t is the unit tax τ_t. Under Assumptions 1-6 and 9, model m is falsified by the instrument if with positive probability over (w_t, z_t), the matrix $[P_m^{-1}P_0]^\star$ has some row whose elements do not sum to 1.*

While the tax rate acts as a cost shifter, there are two important features that make this result different from Corollary 1. First, because the unit tax shifts marginal costs additively, the variation in unit tax rates is not needed to estimate the effect of τ_t on marginal cost, and can therefore be used as instrumental variation. In contrast with other cost shifters, this makes the instrument relevant for falsification even when the true model and the model to be falsified both have diagonal pass-through matrices. However, the unit tax applies to all products equally, rather than shifting marginal costs of one product at a time. For this reason, there may be non-diagonal pass-through matrices for which unit tax instruments do not permit falsification. For example, take $P_{0t} = \left[\begin{smallmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{smallmatrix}\right]$ and $P_{mt} = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$: under either of these two conduct models, a unit increase in the tax rate increases both products' prices by 1, making them observationally equivalent.[19]

Ad Valorem Tax Instruments: Next, we consider the relevance of ad valorem tax instruments. Inspection of Equation (3) shows that while unit taxes have an additive effect on markups (or costs), ad valorem taxes instead have a multiplicative effect on markups. This allows variation in the tax rate to distinguish models with different average *levels* of markups, not only those models with different pass-through matrices. Variation in ad valorem tax rates therefore allows falsification in some cases where it would be impossible with any of the other instruments considered so far.

Example 6: Return to the example environment, and suppose the true model is one in which pre-tax markups are set to a constant level ζ_0 across markets, so that $\Delta_{0jt} = \zeta_{0j}$. We consider falsifying a model in which firms set constant markups $\Delta_{mt} = \zeta_m \neq \zeta_0$. Now, $P_{0t} = P_{mt} = I$. Variation in product characteristics, rival cost shifters, or unit taxes have no effect on markups, making cost side, demand side and unit tax instruments irrelevant for falsification. However, when using ad valorem tax instruments, the falsification condition in Proposition 2 becomes $E\left[\zeta_m - \zeta_0 \mid \mathrm{w}_t, \nu_t\right] \neq 0$, which holds for any $\zeta_m \neq \zeta_0$. Thus, ad valorem taxes are relevant instruments for distinguishing constant markup models. •

This example generalizes to the following corollary:

[19] If the government levied product-specific unit taxes, then falsification would obtain, as $P_{mt} \neq P_{0t}$.

Corollary 4. *Suppose that for each product j, the instrument z_{jt} is ν_t, a transformation of the ad valorem tax rate. Let Assumptions 1-6 and 9 hold. Further suppose that either $P_{mt} = P_{0t}$ or the instrument does not move prices under the true model. Model m is still falsified as long as $E[\Delta_{mt} | \mathrm{w}_t, \nu_t] \neq E[\Delta_{0t} | \mathrm{w}_t, \nu_t]$ w.p.p. over (w_t, ν_t).*

Note that if $P_{mt} \neq P_{0t}$ and the tax rate does move equilibrium prices, then the two terms in Equation (4) are both nonzero, and could theoretically cancel each other out. In fact, this happens for all rule-of-thumb models where after-tax revenue is set to a fixed multiple of (non-tax) marginal costs: if $\nu_t p_{jt} = \phi_m c_{mjt}$, then $\nu_t \Delta_{mjt} = (\phi_m - 1) c_{mjt}$ does not change with ν_t, making falsification of the "wrong" model within this class impossible. For models outside this rule-of-thumb class, the two effects are unlikely to cancel: for example, when comparing Bertrand and Cournot competition, the model with higher markups (Cournot, see Magnolfi, Quint, Sullivan, and Waldfogel, 2022b) is typically associated with lower cost pass-through, which gives the two terms in Equation (4) the same sign, ensuring falsification of the wrong model.

Corollaries 3 and 4, and the discussion above, illustrate the usefulness of tax rates as instruments for conduct. Tax rates enter the equilibrium first-order conditions directly, without introducing additional parameters to be estimated. This enables researchers to use variation in tax rates to distinguish models of conduct even when other instruments fail, or to falsify classes of models that other instruments may not. In addition, as shown in Propositions 1 and 2 , the marginal effect of an instrument on prices plays an important role in its ability to distinguish between models of conduct, and there is a long line of empirical literature documenting the strong effects of tax rate changes on prices (see, e.g., Butters, Sacks, and Seo (2022) and the references therein).[20]

5 Instrument Selection for Counterfactuals

Researchers often want to know the correct model of conduct not for its own sake, but to perform counterfactual analyses. Different counterfactual exercises depend on different features of the model; our falsification framework can identify instruments that target these particular features, making the counterfactuals more credible. As an illustration, we consider the problem of maximizing tax revenue by learning the Laffer curve, which has garnered recent interest (Miravete et al., 2018; Hollenbeck and Uetake, 2021; Brugués and De Simone, 2024).[21]

[20] In contrast, DellaVigna and Gentzkow (2019) provide evidence that some types of demand shifters may only have weak effects on prices in the data-generating process. This also appears to be an issue for other common conduct instruments, leading to inferential problems (Backus et al., 2021; Duarte et al., 2024).

[21] The insights in this section readily extend to other optimal tax policy settings: for example, if the government wished to maximize a convex combination of revenue and consumer surplus.

5.1 Learning the Laffer Curve

Counterfactual predictions are essential to understanding the Laffer curve, as taxes usually have limited support in the data. The policymaker must use the model to forecast revenue at unobserved tax rates. We limit attention to the optimal tax in a single market, t, for expositional parsimony. Government revenues in market t for any tax rates (ν, τ) are

$$R_t(\nu,\tau) = \tau \sum_j s_{jt}\big(p_{0t}(\nu,\tau)\big) + (1-\nu)\sum_j p_{0jt}(\nu,\tau)s_{jt}\big(p_{0t}(\nu,\tau)\big). \tag{5}$$

This expression emphasizes that market shares depend on prices, and prices on tax rates; all other primitives are held constant and suppressed. The observed price $p_t = p_{0t}(\nu_t, \tau_t)$ corresponds to the current tax rates (ν_t, τ_t). The policymaker knows the revenue at observed tax rates, but must use a model m to predict counterfactual prices $p_{mt}(\nu, \tau)$ and therefore revenue at other rates. Recent literature (Miravete et al., 2018; Hollenbeck and Uetake, 2021) shows that the Laffer curve can differ substantially across different models. The policymaker, not observing the true conduct model, wants the model that best matches the true Laffer curve. Our framework sheds light on the link between falsification by particular instruments and the counterfactual Laffer curve.

We assume the government optimizes one tax rate, keeping the other fixed. Our application to the Washington cannabis market exemplifies this: the government only uses an ad valorem tax. We analyze the Laffer curves for unit and ad valorem tax rates separately.

Unit Tax: Fix the ad valorem tax at its observed level $\nu = \nu_t$ and allow the unit tax to vary. Equation (5) reveals which aspect of firm conduct determines the Laffer curve. Assuming the demand function $s_{jt}(\cdot)$ is identified, we need only identify $p_{0t}(\nu_t, \cdot)$, the effect of unit tax on prices. Applying the Implicit Function Theorem to Equation (3) yields $\frac{\mathrm{d}p_{0t}}{\mathrm{d}\tau} = \frac{1}{\nu_t} P_{0t}(\tau)\iota$, where ι is a J-vector of ones.[22]

Corollary 5. *Under Assumptions 1-6 and 9, if $P_{mt}\iota = P_{0t}\iota$ everywhere, then model m produces the true Laffer curve for the unit tax.*

The proof, detailed in the Appendix A, can be summarized as follows. All models are constructed to rationalize observed prices, so $p_{mt}(\tau_t) = p_{0t}(\tau_t) = p_t$. With ν_t fixed, $P_{mt}\iota = P_{0t}\iota$

[22] Formally, stack Equation (3) and rewrite it as $\nu_t p_t - \nu_t \Delta_{0t} - \tau_t \iota = 0$. With ν_t fixed, define $F(p_t, \tau_t)$ as the left-hand side, so $p_{0t}(\tau)$ is implicitly defined as the solution to $F(p_{0t}(\tau), \tau) = 0$. The Implicit Function Theorem then gives $\frac{\mathrm{d}p_{0t}}{\mathrm{d}\tau} = -\left(\frac{\partial F}{\partial p_t}\right)^{-1} \frac{\partial F}{\partial \tau_t} = -\left(\nu_t I - \nu_t \frac{\partial \Delta_{0t}}{\partial p_t}\right)^{-1}(-\iota)$ under Assumption 9. As shown in the text, $P_{0t} = \left(I - \frac{\partial \Delta_{0t}}{\partial p_t}\right)^{-1}$, and therefore $\frac{\mathrm{d}p_{0t}}{\mathrm{d}\tau_t} = \frac{1}{\nu_t} P_{0t}\iota$.

is sufficient for the derivatives of $p_{mt}(\cdot)$ and $p_{0t}(\cdot)$ to match everywhere, making these functions identical and resulting in identical Laffer curves. Thus, Corollary 5 shows that model m yields the correct optimal tax when the row sums of the model's pass-through matrix equals that of the data-generating pass-through matrix, a weaker requirement than $P_{mt} = P_{0t}$.

Using the unit tax as an instrument creates a close link between the falsification condition in Proposition 2 and the Laffer curve equivalence condition in Corollary 5. With a single unit tax applying to all products, the falsification condition in Equation (4) simplifies to: $E\left[P_{mt}^{-1}\left(P_{0t}\iota - P_{mt}\iota\right) \mid \mathrm{w}_t, \tau_t\right] \neq 0$. Therefore, model m is not falsified when $P_{mt}\iota = P_{0t}\iota$ almost surely on the data's support. Thus, using the unit tax as the instrument directly targets the most relevant part of the pass-through matrix for the unit tax policy counterfactual. Example 7 further illustrates this connection.

Example 7: Consider the simple environment of Example 1 where the true model is Keystone pricing and the alternative model is Bertrand. Under Proposition 2, variation in a unit tax, which moves own and rival cost, falsifies Bertrand as $P_{Bt}\iota < 1 < 2 = P_{Kt}\iota$. Further, since $P_{Bt}\iota \neq P_{Kt}\iota$, the Laffer curve under Bertrand differs from the truth. Now consider falsifying a shifted Keystone model ($m = S$) whereby $p_{jt} = 2c_{jt} + \zeta_{jt}$, where ζ_{jt} is a constant. Notice that, since $P_{St} = P_{Kt}$ and therefore $P_{St}\iota = P_{Kt}\iota$, the shifted Keystone model is not falsified by the unit tax instrument. However, because the pass-through matrices are the same, the two models imply the same unit tax Laffer curve. Thus, while variation in the unit tax may not falsify all wrong models, it will permit the researcher to learn the true Laffer curve. •

Ad Valorem Tax: Now fixing the unit tax at $\tau = \tau_t$, we examine the Laffer curve for the ad valorem tax. With demand identified, accurately capturing the tax's effect on prices is once again the key step. Applying the Implicit Function Theorem to Equation (3) yields $\frac{\mathrm{d}p_{0t}}{\mathrm{d}\nu} = \frac{1}{\nu}P_{0t}(\nu)(\Delta_{0t}(\nu) - p_{0t}(\nu))$, where only ν varies and we suppress dependence on other arguments. The key term to determine the Laffer curve is $P_{0t}(\Delta_{0t} - p_{0t})$. This resembles the unit tax case, with $\Delta_{0t} - p_{0t}$ replacing ι due to ν_t's multiplicative entry in the equilibrium condition. As before, imposing model m will result in a potentially different implied Laffer curve.

Corollary 6. *Under Assumptions 1-6 and 9, if $P_{mt}(\Delta_{mt} - p_{mt}) = P_{0t}(\Delta_{0t} - p_{0t})$ whenever $p_{mt} = p_{0t}$, then model m produces the true Laffer curve for the ad valorem tax.*

A consequence of Corollary 6 is that any rule-of-thumb model where markups are a constant fraction of prices – such as Keystone and marginal cost pricing – will yield the same Laffer curve.[23] For model m in the rule-of-thumb class, $\Delta_{mt} = \psi_m p_t$, and $P_{mt} = \frac{1}{1-\psi_m}I$ and $(\Delta_{mt} - p_t) = (\psi_m - 1)p_t$ so that $P_{mt}\left(\Delta_{mt} - p_t\right) = -p_t$, which does not depend on ψ_m.

[23]For Keystone, $\Delta_{Kt} = 0.5p_t$, while for marginal cost pricing, $\Delta_{MCt} = 0 \times p_t$.

Using the ad valorem tax as a conduct instrument links falsification to the optimal tax counterfactual. Because $p_t = p_{0t} = p_{mt}$ in the data, the falsification condition in Equation (4) simplifies to $E\left[P_{mt}^{-1}\bigg(P_{0t}\left(\Delta_{0t} - p_t\right) - P_{mt}\left(\Delta_{mt} - p_t\right)\bigg) \mid \mathrm{w}_t, \nu_t\right] \neq 0$. If $P_{mt}\left(\Delta_{mt} - p_t\right) = P_{0t}\left(\Delta_{0t} - p_t\right)$ almost surely in the support of the data, model m is not falsified. Thus, the ad valorem tax instrument targets the most relevant objects for the tax counterfactual.

5.2 Counterfactual-Relevant Instruments: A Discussion

Corollaries 5 and 6 show that the tax rate itself is the most relevant instrument to learn conduct to find the Laffer curve, as it targets the most crucial aspects of pass-through and markups. However, this raises a question: if we observe both tax rates and market outcomes, why use a structural model instead of simply identifying the tax rate with the highest observed revenue? In real-world data, variation in observed tax rates is often limited, while finding the optimal tax requires inferring revenue for every potential tax rate. Using the existing variation in taxes allows us to learn the model of conduct that best fits the features of the data relevant to the Laffer curve. However, when the data does not contain any variation in the tax rate of interest, a researcher can use our framework to understand the tradeoffs involved in selecting another instrument (i.e., the other tax rate, rival cost shifters, or product characteristics) to learn conduct and subsequently the Laffer curve.

To illustrate how our framework informs selecting "second best" instruments, consider the case of setting an optimal unit tax. The key object is $P_{0t}\iota$, which the unit tax instrument directly targets. Insofar as variation in the ad valorem tax rate exists, it may seem intuitive to use variation in the "other tax" to learn conduct and the Laffer curve. However, our results above show that the ad valorem tax targets $P_{0t}\left(\Delta_{0t} - p_{0t}\right)$, making it generally difficult to target $P_{0t}\iota$ with these instruments. In the special case where marginal costs are the same for all products, so that $c_{0jt} = c^*_{0t}$ and $c_{mjt} = c^*_{mt}$ for all (j,t), falsifiability requires $E\left[\frac{1}{\nu_t^2} P_{mt}^{-1}\left(c^*_{0t} P_{0t}\iota - c^*_{mt} P_{mt}\iota\right) \mid \mathrm{w}_t, \nu_t\right] \neq 0$. At best, the ad valorem tax instrument can only target $P_{0t}\iota$ up to an unknown, market-varying scale factor c^*_{0t}. Thus, it appears that the ad valorem tax instrument cannot be first-best for the unit tax Laffer curve.

With demand side and cost side instruments, the situation is more optimistic, though there are tradeoffs. While our results in Section 4 show that demand and cost side instruments do not generally target $P_{0t}\iota$, cases exist where both target P_{0t} directly. Outside of the knife-edge case where the indirect and direct effects perfectly offset, demand side instruments constructed with own and rival product characteristics simultaneously target both P_{0t} and the direct effect. Furthermore, when the demand system satisfies Assumption 8 and the conduct model satisfies Assumption 7, the demand side instruments target only P_{0t}. For cost side instruments, consider the special case where marginal cost depends additively a special shifter w^*_{jt} such that $c_{jt} = \bar{c}_{jt} + \gamma \mathrm{w}^*_{jt} + \omega_{0jt}$. When a researcher both observes w^* and knows the value of γ (e.g., for observed wholesale prices, $\gamma = 1$), they can use variation in own and rival w^* to construct instruments. These instruments function like product-specific unit taxes, and it is straightforward to show that they target P_{0t}.

The benefit of targeting P_{0t} is that models where $P_{mt} = P_{0t}$ will deliver the true Laffer curve. The downside is that targeting the full pass-through matrix P_{0t} is stricter than targeting $P_{0t}\iota$. This is potentially problematic because it implies that such instruments may falsify models which deliver the true Laffer curve. One can circumvent this issue by constructing an instrument from the market-level average of x_{jt} or w^*_{jt} (across own and rival products). Under Assumptions 7 and 8, using the average value of product characteristics $\bar{x}_t$ as an instrument yields the falsification condition $\frac{\beta}{\alpha} E\left[P_{mt}^{-1}\left(P_{0t}\iota - P_{mt}\iota\right) \mid \mathrm{w}_t, \tilde{x}_t, \bar{x}_t\right] \neq 0$, where $\tilde{x}_{jt} = x_{jt} - \bar{x}_t$. Similarly, when the special shifter w^*_{jt} is observed and γ is known, using $\bar{\mathrm{w}}^*_t$ as the instrument directly targets $P_{0t}\iota$ as the falsification condition becomes $\gamma E\left[P_{mt}^{-1}\left(P_{0t}\iota - P_{mt}\iota\right) \mid \tilde{\mathrm{w}}_t, \bar{\mathrm{w}}_t\right] \neq 0$. In either case, the market-level average instrument targets $P_{0t}\iota$ directly.

Consider now the case of setting an optimal ad valorem tax. Both pass-through and the level of markups determine the ad valorem Laffer curve, and no instrument other than ad valorem taxes targets both of these objects. Which instrument is second-best in this case depends on the relative importance of indirect and level effects, which will vary by application.

Overall, these examples show how researchers can use our falsification framework to understand the tradeoffs inherent to various instruments. In the case of Laffer curves, imposing additional assumptions – e.g., linearity, known coefficients, and index restrictions – can enable researchers to effectively replicate the first-best conduct instrument. These assumptions can be supported by economic intuition and institutional knowledge, and in some cases they are testable.[24] Whether such a replication is possible depends on the counterfactual at hand.

6 Monte Carlo Simulations

We further illustrate the main results of the paper with simulations tied to the examples described in the preceding sections. To align the environment with the falsification framework, we simulate data for 50,000 markets using the simulation class in `PyBLP` (Conlon and Gortmaker, 2020). In each market t, the number of single-product firms J_t is a randomly chosen integer between two to ten, leaving us with 319,719 observations in the sample. We adopt a simple logit demand system, in line with our falsification examples.

The parameterization of demand results in a mean own price elasticity of -8.59 and

[24] For example, the demand side index restriction in Assumption 8 is testable.

diversion to the outside option of 0.37. On the supply side, we assume that the marginal costs are a linear function of two observed cost shifters, which are excluded from demand, and of ω_{0jt}, the true unobserved cost shock. The government levies both a unit tax (τ_t) and an ad valorem tax (υ_t) on all products in market t. The unit tax is remitted by the firms while consumers remit the ad valorem tax. We assume that the true model of conduct is Keystone pricing, whereby firms set tax exclusive prices as twice their marginal cost, or $\nu_t p_{jt} = 2c_{jt} + 2\tau_t$. Full details of our simulation environment are available in Appendix D.

6.1 Testing with Rivers and Vuong (2002)

The researcher considers a menu of four models: the true Keystone model ($m = K$), Bertrand ($m = B$), Cournot ($m = C$), and a shifted Keystone model ($m = S$). For the shifted Keystone model, $\nu_t p_{jt} = \nu_t \eta + 2c_{mjt} + 2\tau_t$, where a fixed $\eta = 1$ is added to markups for all products in all markets. To make inferences on firm conduct in finite samples, the researcher adopts the model selection test in Rivers and Vuong (2002) (RV). For the RV test, the null hypothesis states that two competing models of conduct $m = 1, 2$ have the same lack-of-fit, or $H_0^{\text{RV}} : Q_1 = Q_2$, while the alternatives correspond to superior fit of one of the two models, $H_1^{\text{RV}} : Q_1 < Q_2$ and $H_2^{\text{RV}} : Q_2 < Q_1$. With this formulation of the hypotheses, the testing procedure determines which of the two models has the smallest lack of fit.

As in Duarte et al. (2024), we express Q_m as a GMM objective function $Q_m = g_m' W g_m$, where $g_m = E[z_{jt}\omega_{mjt}]$ and $W = E[z_{jt}z_{jt}']^{-1}$ is the weight matrix. For this GMM measure of fit, the RV test statistic is then

$$T^{\text{RV}} = \frac{\sqrt{n}(\hat{Q}_1 - \hat{Q}_2)}{\hat{\sigma}_{\text{RV}}},$$

where $\hat{Q}_m = \hat{g}_m' \hat{W} \hat{g}_m$, $\hat{g}_m = n^{-1}\hat{z}'\hat{\omega}_m$, $\hat{W} = n(\hat{z}'\hat{z})^{-1}$, and $\hat{\sigma}^2_{\text{RV}}$ is an estimator for the asymptotic variance of $\sqrt{n}(\hat{Q}_1 - \hat{Q}_2)$.[25]

By imposing the true demand system, and considering the true model as one of the alternatives, the RV test has a close connection to the falsification framework. Suppose the true model is model 1; then $\hat{Q}_1 \simeq 0$ in large samples. Then, the numerator of the RV test coincides with the falsifiable restriction for model 2, and rejection of the RV null in favor of model 1 coincides with the falsification of model 2. However, when both models are not falsified, the RV test statistic is degenerate, causing severe inferential problems. Thus, the $F-$statistic diagnostic from Duarte et al. (2024) can be used as finite-sample evidence of instrument relevance. In addition to this ex-post diagnostic, the results in Sections 4 and

[25]See Duarte et al. (2024) for a variance estimator accounting for randomness in $\hat{W}$ and $\hat{g}_m$.

5 show how a researcher can use the economics of pass-through implied by the models of conduct being tested to choose ex-ante potentially relevant instruments.

To perform testing, we consider four sets of instruments: the unit tax τ_t, the ad valorem tax v_t, cost side instruments (the sum of rival cost shifters in a market, denoted as "Riv. Cost"), and demand side instruments (the number of rival products in a market and the sum of rival product characteristics, denoted as "Riv. BLP"), which we also pair with own characteristics ("Riv. BLP + Own x's"). We run the RV test separately with each instrument set.

6.2 Simulation Results

Test Results: Table 1 presents the results of testing all pairs of models. Examples 1-6 suggest that when testing Bertrand against Keystone, all instruments should be relevant.[26]

TABLE 1: Test Results for Simulated Data

Statistic	Instruments: Unit Tax (1)	Ad Valorem Tax (2)	Riv. Cost (3)	Riv. BLP (4)	Riv. BLP + Own x's (5)
Panel A: Bertrand vs. Keystone					
T^{RV}	25.2	16.1	9.8	16.3	29.4
	$***$	$***$	$***$	$***$	$***$
F	249.3	138.1	21.2	44.8	157.6
	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$
Panel B: Cournot vs. Keystone					
T^{RV}	23.7	23.4	-0.0	0.7	40.8
	$***$	$***$			$***$
F	47.8	46.8	0.1	0.0	66.8
	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$	$\dagger\dagger\dagger$	$\dagger\dagger\dagger$	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$
Panel C: Shifted Keystone vs. Keystone					
T^{RV}	-0.1	4.8	0.6	0.5	0.5
		$***$			

The table reports, for each set of instruments and pair of models, the RV test statistics T^{RV}. Panels A and B also report the effective F-statistic (Duarte et al., 2024); we don't report it in Panel C as the pair of models does not satisfy Assumption 2 in Duarte et al. (2024). Columns 1-5 correspond to different instruments; panels A-C correspond to three different pairs of models. A positive RV test statistic suggests a better fit of Keystone. The symbol $***$ indicates rejection of the null of equal fit 0.01 confidence level. The symbols $\dagger\dagger\dagger$ and $\wedge\wedge\wedge$ indicated that F is above the appropriate critical values for worst-case size below 0.075, and best-case power above 0.95, respectively. Both T^{RV} and the F-statistics account for market level clustering. $n = 319,719$.

We see in Panel A that the RV test statistics are all positive for this pair of models and are well above 1.96, so that the null is rejected in favor of Keystone. When testing Bertrand

[26] A table in Appendix E summarizes instrument relevance results for these examples.

against Keystone, the F-statistics are all larger than the critical values for worst-case size of 0.075 and best-case power of 0.95, suggesting all instruments are relevant.

By contrast, Example 3 suggests that variation in rival product characteristics and rival cost shifters are irrelevant for falsifying Cournot against Keystone when demand is logit. We see in Columns 3-4 of Panel B that these instruments have virtually no power and the null is not rejected. Example 4 however suggests that variation in own product characteristics, which can pick up on differences in the main diagonal of the pass-through matrices, should falsify Cournot. In Column 5, we see that including own product characteristics creates relevant instruments which reject the null in favor of the true Keystone model. Examples 5 and 6 suggest that the tax instruments are relevant: Columns 1-2 show that these instruments are strong for size and power and reject the null in favor of Keystone.

Finally, when testing the shifted Keystone model against Keystone (Panel C), both models have the same pass-through matrices as discussed in Example 7. Thus, we expect the unit tax, product characteristics (own and rival), and rival cost shifters (Columns 1,3-5) to be irrelevant, and T^{RV} is close to zero in those cases. However, the level of the markups differs across the two models. Ad valorem tax instruments (Column 2), which can falsify a wrong model based on the level effect, strongly reject the shifted Keystone model in favor of Keystone.

Laffer Curve: Next, we consider the tax implications of the models tested above. To mimic the problem facing a state government setting a single tax rate across many markets, we randomly select 500 of the 50,000 markets in our simulated data. In Panel A of Figure 1, we compute the Laffer curve for the unit tax, holding the ad valorem tax fixed at the levels in the simulated data. To obtain the Laffer curve under a given model of conduct, we first back out c_{mjt}, the implied marginal cost under that model. Holding demand and cost fixed, we then solve for equilibrium at each level of the tax and compute the associated revenue.

From the results in Table 1, we find that Keystone and shifted Keystone are the two models that are not falsified by unit tax instruments, as they feature the same pass-through matrices. Furthermore, in line with Corollary 5, we see in Panel A of Figure 1 that these models imply the same Laffer curve. Conversely, since Bertrand and Cournot yield different Laffer curves than the true one, they have different $P_{mt}\iota$ than Keystone, which permits the falsification of these models with unit tax instruments (Panels A and B of Table 1).

In Panel B of Figure 1, we compute the Laffer curve for the ad valorem tax, holding the unit tax fixed at their simulated levels and following the same steps outlined for Panel A. In Table 1, we find that Keystone is the only model not falsified by ad valorem tax instruments. This is in line with Corollary 6: the other models all produce a different Laffer curve, thus have different $P_{mt}(\Delta_{mt} - p_t)$, and are therefore falsified by ad valorem tax instruments.

These results illustrate three ways in which our falsification framework can be used. First,

Figure 1: Laffer Curve for Unit and Ad Valorem Taxes

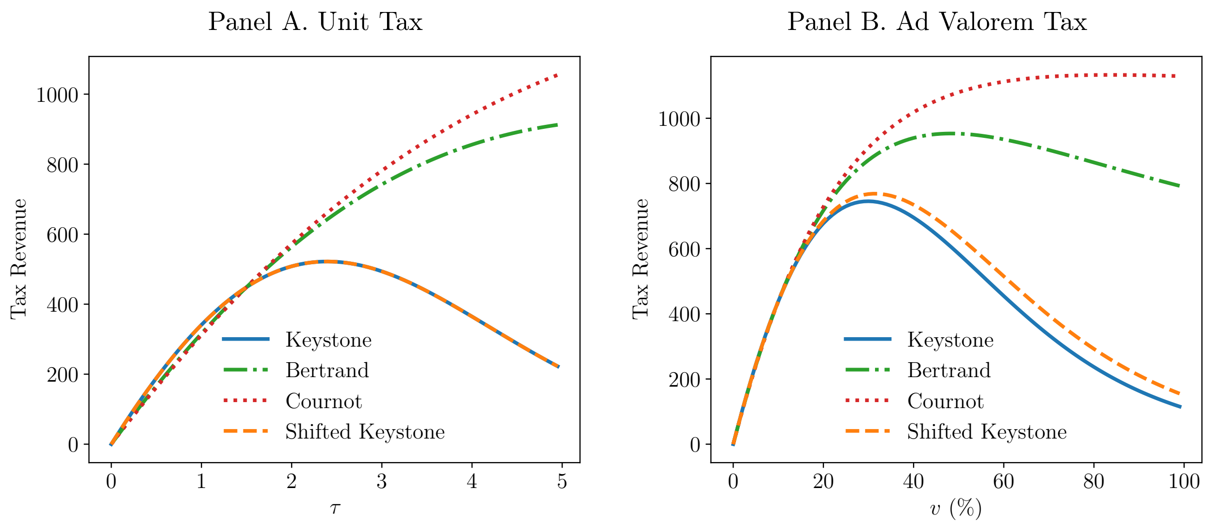


Figure illustrates the Laffer curve implied by different models of conduct. In Panel A, we hold fixed the ad valorem tax in each market and vary the unit tax from 0 to 5. In Panel B, we hold fixed the unit tax but vary the ad valorem tax from zero to one. To compute the curves in each panel, we simulate counterfactual prices, quantities, and government revenue at different levels of the appropriate tax for each candidate model.

the falsification results in Section 4 enable ex-ante instrument selection to avoid irrelevant instruments. For instance, when testing Keystone versus Cournot, our results suggested that cost side instruments would be irrelevant. Second, our framework can motivate instrument choice for specific models. For instance, we found ad valorem tax instruments to be particularly useful to falsify the shifted Keystone model, which is otherwise hard to distinguish from Keystone. Finally, when a researcher targets a specific counterfactual, as in Section 5, they can use our framework to tailor the instrument choice for learning the optimal policy.

With these takeaways in mind, we now turn to an empirical setting where conduct is unknown, and policymakers are interested in setting revenue-maximizing taxes.

7 Empirical Application

As an application of our results, we consider the problem of setting the optimal ad valorem tax in Washington State's marijuana market, which is well-suited for several reasons. First, setting the optimal tax is an important consideration in this market, as revenues are substantial. Second, optimal taxes depend on conduct (Miravete et al., 2018; Hollenbeck and Uetake, 2021), and it is unclear how well retailers' conduct in this market aligns with standard structural models (Escudero, 2018; Hollenbeck et al., 2024). Third, our results suggest that for a tax exercise, conduct would best be learned using an ad valorem tax instrument, which is available in this market through local cannabis tax variation.

7.1 Institutional Background and Data

Washington legalized cannabis for medical use in 1998, and in 2012, through voter approval and passage of I-502, was among the first states to legalize recreational cannabis.[27] In July of 2014, retail sales of cannabis began in Washington.

The state regulates the cannabis industry in several ways. First, there is a cap on the number of retailers.[28] Second, the state restricts vertical relations between producers (growers), processors, and retailers. While producers and processors — those who convert marijuana plants into various products — can integrate with one another, neither can integrate with a retailer. Only retailers can sell cannabis products directly to consumers. Third, the state assesses an ad valorem tax on cannabis sales. When cannabis was first legalized, the state levied a 25% tax at each point of sale — from producers to processors, processors to retailers, and retailers to consumers. In July 2015, this changed to a 37% tax on the final sale to consumers only.[29] Additionally, local municipalities imposed their own ad valorem taxes, which range from seven to ten percent.

In part to enforce tax collection, the state tracks detailed "seed-to-sale" data on all cannabis products. Our empirical application benefits from these unusually detailed data. We construct a dataset containing product shares and prices for many markets. The underlying data come from BioTrack, an administrative data collection software that tracks cannabis from plant production and processing through retail ("seed-to-sale").[30] These data include the universe of wholesaler and retailer transactions for cannabis products. Thus, for every transaction, we observe prices paid both by consumers to retailers as well as by retailers to wholesalers. Biotrack also collects supplemental information on the organizations in the market, from which we observe when retailers began operating, and lab testing of plants, from which we observe cannabis product CBD and THC content.

Due to the difficulty of comparing package size weights between different product types (e.g. liquid, extracts, and solids), we only keep products labeled as "usable" marijuana (dried leaves and flowers), which represent 72% of all transactions in the data. Within this category, we subset to package sizes of 1 and 3.5 grams (following Escudero, 2018), which are by far the most popular, accounting for 60% of usable product revenue.[31] Features of the cannabis

[27] As of 2024, 38 states have legalized cannabis for medical use and 24 for recreational use.

[28] The Washington State Liquor and Cannabis Board initially limited the number of retailer licenses to 334, and allocated licenses across counties proportional to population. In jurisdictions with too many applications, licenses were assigned via lottery. In January 2016, the state increased the number of retail licenses to 556. The state also caps the number of each licensee's retail locations at three.

[29] Hansen, Miller, and Weber (2022) explore the effects of this tax change on the tendency for producers and processors to vertically integrate.

[30] These tracking efforts are, in part, to thwart sales in neighboring states where cannabis was illegal.

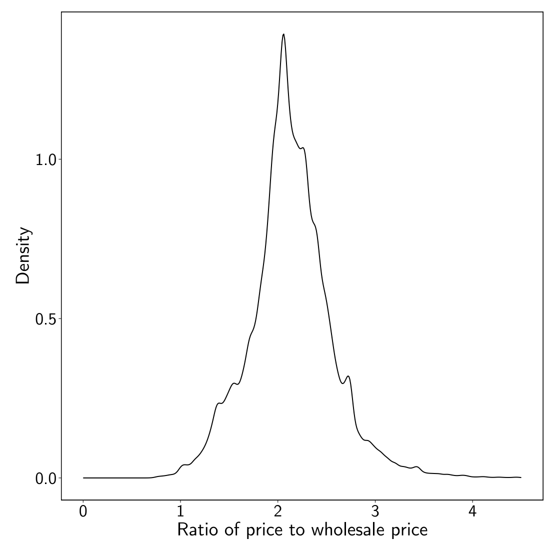
[31] See Table 4 in Appendix F for a summary of all data cleaning steps.

market complicate the observed prices in the raw data. Because cannabis is illegal at the federal level, retailers accept cash payments and manually record transactions, resulting in inconsistencies regarding whether reported prices included the tax. Hollenbeck and Uetake (2021) correct for this, and we use their reconstructed tax-inclusive retail prices.[32]

We define a product to be a package size from a processor sold at a retailer in a month and markets to be city-month pairs. We calculate a product's price as the sales volume-weighted average price. We define the market size to be twice the maximum amount ever purchased in a given geographic market. We further draw 2000 consumers' incomes from the empirical distribution of income in each city, which we obtain from the US Census. The data include prices and shares for 1,428 products across 2,639 markets from August 2014 to May 2017.

Appendix Table 5 displays summary statistics of the data used in estimation. Wholesalers charge an average of \$3.60 for a gram of cannabis, while retailers charge \$7.60, on average. While the average market has over 100 cannabis processors, retail market quotas restrict the average market to 14 retailers. We follow the literature (Escudero, 2018; Hollenbeck and Uetake, 2021) in concluding from these facts that wholesaler market power is unlikely to be important, and we focus our attention on how retailers set prices instead.

FIGURE 2: Ratio of Retail and Wholesale Price



The figure plots densities of the ratio of retail and wholesale prices for each product in our data.

Figure 2 graphs the ratio of retail unit prices to wholesale unit prices. We see a mass of ratios concentrated around two, lending credence to the hypothesis that firms engage in Keystone pricing. A formal test follows in Section 7.3.

[32]Additional details on data construction can be found in Appendix A of Hollenbeck and Uetake (2021).

7.2 Demand Estimation

We follow Hollenbeck and Uetake (2021) in adopting a random coefficient nested logit demand model (Grigolon and Verboven, 2014). We briefly summarize our demand model here, and provide a more detailed exposition in Appendix F.

Product characteristics include a constant, package size, THC and CBD levels (and their values squared), and the log of the number of products offered in the store to capture variation in shelf space across stores. We allow consumer preferences for characteristics and prices to vary with individual-level income. We also include fixed effects at the retailer, processor, and year-month levels. Our nesting structure includes all inside goods in one nest.

Following Hollenbeck and Uetake (2021) we construct demand instruments from several sources of exogeneous variation. These include the number of products sold at competing dispensaries in the market, also interacted with the mean income, BLP-style instruments (sum of THC and CBD for products within a store, and for products in all other stores), and exogenous own cost shifters (rainfall and temperature in the region of production and their lags). We estimate demand using `PyBLP` (Conlon and Gortmaker, 2020).

Results for demand estimation are reported in Appendix Table 6. Similar to Hollenbeck and Uetake (2021), we find a positive interaction between income and price and a negative interaction between income and the constant. We also find that consumers prefer products with a higher level of THC, though at a declining rate, while they dislike products with higher levels of CBD. The median own-price elasticity in our sample is -6.451 suggesting a fair degree of substitution across strands of usable marijuana. Inspection of the aggregate elasticity allows comparison to the results in Hollenbeck and Uetake (2021), who aggregate products of the same type within a store, and to demand for related products (e.g., liquor). Our aggregate elasticity of -3.32 (-2.9 when weighted by market size) is in line with median own-price elasticity in Hollenbeck and Uetake (2021) (-2.89) and broadly similar to the aggregate elasticity in the Pennsylvania liquor market found in Miravete, Seim, and Thurk (2020) (-2.48).

One potential concern is that our implementation of the RCNL model restricts the range of curvature of demand (Miravete et al., 2024). In turn, misspecifying demand in this way can lead to restrictions on the pass-through implied by certain models of conduct (e.g., Bertrand). Because we show, in the previous section, that differences in pass-through matrices help distinguish different models of conduct, this potential misspecification could affect our test results. Miravete et al. (2024) allow for greater flexibility in modeling heterogeneous price sensitivity by incorporating a Box-Cox transformation of the income interaction. They show that this can greatly increase the range of curvature implied by discrete choice demand models. Without micro-data, we lack sufficient identifying power to pin down the additional parameters required by this approach. Instead, in Appendix G, we calibrate different approximations for the Box-Cox transformations for income and include them in the RCNL demand system. We find that our testing results are unaffected.

7.3 Test for Conduct

Models of Conduct: We consider three models of conduct that are discussed in previous literature for this market: (i) *Marginal Cost Pricing*—retailers choose tax-exclusive prices equal to marginal cost; (ii) *Keystone Pricing*—retailers choose tax-exclusive prices equal to twice marginal cost; and (iii) *Bertrand Pricing (Nash Price Setting)*—retailers set prices competitively to maximize profits in the complete information pricing game (considered in Hollenbeck and Uetake, 2021). Escudero (2018) provides some descriptive evidence that Keystone pricing is plausible in the market during our sample period, while Hollenbeck et al. (2024) provide evidence of strategic interactions in a later period. It is thus unclear *a priori* which model best fits the data during our sample period.

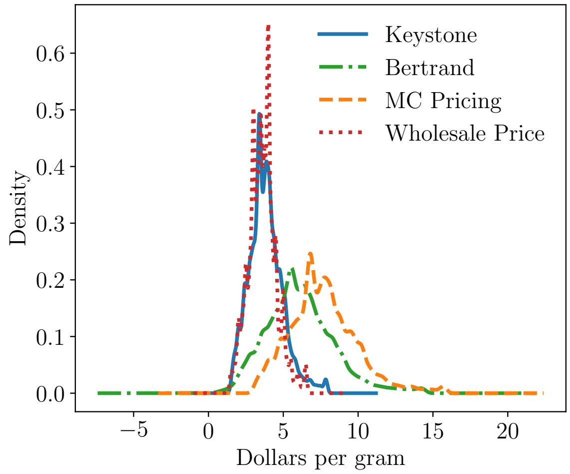
For the first two models, implied costs and markups can be immediately computed from observed prices. Given our demand estimates, we can also compute Δ_B and c_B for the Bertrand model. We specify marginal cost as the sum of observed wholesale prices p^W and additional retail costs. In turn, these include a linear function of observed shifters w and an unobserved shock. The vector w_{jt} includes product fixed effects.[33]

Inspection of Implied Costs and Markups: Before performing formal testing, we inspect the data and the implications of different models for markups and costs. Given that we observe wholesale prices, and that these likely make up a large fraction of the marginal cost of selling a product, it is natural to compare the model-implied costs to these prices. Figure 3 reports the distributions of implied costs for all models, as well as the observed wholesale prices in dollars per gram. We notice first that Keystone implies lower marginal costs than Bertrand which in turn implies lower marginal costs than marginal cost pricing. While the distribution of the Keystone implied costs is close to the distribution of wholesale prices, the other models' implied cost distributions are far from wholesale prices.

The descriptive statistics reported in Table 2 further shed light on the differences across the three models. The fit as measured by RMSE of implied costs with respect to observed wholesale prices is substantially better for Keystone. Moreover, wholesale prices represent on average 98% of the Keystone total marginal cost. In contrast, marginal cost pricing and Bertrand imply that wholesale prices represents only 49% and 65% of retail marginal costs, respectively. Inspection of markups reveals that while firms have market power under Bertrand,

[33] In Appendix G, we show the robustness of our test results to alternative specifications of cost.

Figure 3: Model Implied Marginal Costs



Distributions of marginal costs implied by different models of conduct along with observed wholesale prices

markups are even higher under Keystone pricing. While not conclusive, our descriptive analysis suggests that the Keystone model is the most reasonable of the three and that the model of marginal cost pricing is implausible in this market. Additionally, our findings in Section 5.1 indicate that both Keystone and marginal cost pricing will produce the same counterfactual Laffer curve for the ad valorem tax because they have identical ad valorem tax pass-through. Thus, going forward, we focus on distinguishing Bertrand from Keystone.

Table 2: Descriptive Statistics of Model Implied Markups and Costs

Statistic	MC Pricing	Keystone	Bertrand
RMSE of c_m and p^W	4.38	0.78	3.04
Mean p^W/c_m	0.49	0.98	0.65
Mean Δ_m/p	0	0.50	0.22
Mean Δ_m (\$)	0	3.84	1.54
25th percentile Δ_m (\$)	0	3.10	1.20
50th percentile Δ_m (\$)	0	3.75	1.30
75th percentile Δ_m (\$)	0	4.45	1.69

We report descriptive statistics of measures of model implied markups and costs. Each column corresponds to a different model of conduct.

Implementation of the Test: To make inference on firm conduct in finite samples, we adopt the model selection test in Rivers and Vuong (2002) (RV) introduced in Section 6, which requires instruments. Because we have variation in the data in state-wide and local ad valorem taxes, our results in Section 5 suggest that for an optimal ad valorem taxation counterfactual, we should use ad valorem tax instruments. However, if the researcher has a different objective, such as learning conduct for its own sake, they may want to consider different sets of relevant instruments. In addition to taxes, three other sources of variation in the data should be relevant for testing. These are (i) rival cost shifters - RC (sum of rainfall, temperature, and their lags); (ii) product characteristics - PC (own and rival number of products, total own and rival CBD and THC); (iii) wholesale prices - WP. In our setting, wholesale prices are functionally equivalent to product-specific unit tax rates.

While all of these instruments are exogenous under standard assumptions (including marginal cost pricing upstream), their relevance can be analyzed in light of our falsification framework. Assuming that the true model is one of the two candidates we test in this application, any of these instruments is plausibly relevant, i.e. capable of falsifying the wrong model, as shown in the previous examples. However, we also learn that different instruments leverage different features of the pass-through matrix to distinguish models of conduct. Thus, using each set of instruments separately to perform the test will provide robust evidence.

Beyond our falsification results, the ability of different sets of instruments to falsify conduct in a finite sample will depend on the variation present in the data. To assess the strength of instruments in the data, we report values of the F-statistic for the RV test developed in Duarte et al. (2024).

Test Results: We test the Bertrand and Keystone model using the RV test with Tax, RC, PC, and WP instruments. We report the results in Table 3.[34]

TABLE 3: Test Results

	Instruments:			
	(Preferred)	(Other Instruments)		
Statistic	Tax	RC	PC	WP
	(1)	(2)	(3)	(4)
T^{RV}	13.39	4.52	9.88	16.79
	$***$	$***$	$***$	$***$
F	40.6	2.2	15.6	1,591.6
	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$	$\dagger\dagger\dagger$ $\wedge\wedge\wedge$

The table reports, for each set of instruments, the RV test statistics T^{RV} and the effective F-statistic (Duarte et al., 2024) for testing Bertrand versus Keystone. A positive RV test statistic suggests a better fit of Keystone. The symbol $***$ indicates rejection of the null of equal fit 0.01 confidence level. The symbols $\dagger\dagger\dagger$ and $\wedge\wedge\wedge$ indicated that F is above the appropriate critical values for worst-case size below 0.075, and best-case power above 0.95, respectively. Both T^{RV} and the F-statistics account for two-step estimation error and clustering at the market level. $n = 153,936$.

In each column – corresponding to a set of instruments – the table reports the RV test statistic T^{RV} and the effective F-statistic (Duarte et al., 2024) for testing Bertrand versus

[34]Results are obtained with `pyRVtest` (Duarte, Magnolfi, Sølvsten, Sullivan, and Tarascina, 2022).

Keystone. A positive value of the RV test statistic above 1.96 indicates rejection of the null of equal fit in favor of better fit for Keystone at the 0.05 confidence level. All sets of instruments decisively reject the null in favor of Keystone. Given that we observe rejection, we are not concerned by low power, but we may still be concerned that these results are due to weak instruments leading to size distortions. For all instruments, F is above the critical value for the worst-case size of 0.075, suggesting size distortions are likely to be minimal.

Table 3 provides strong evidence that Keystone better describes conduct in this market. Across sets of instruments, each leveraging different features of the two models, the data consistently support Keystone as the superior model of conduct during our sample period, compared to Bertrand. In the next section, we explore how performing counterfactuals under the Keystone model has important implications for public policy.

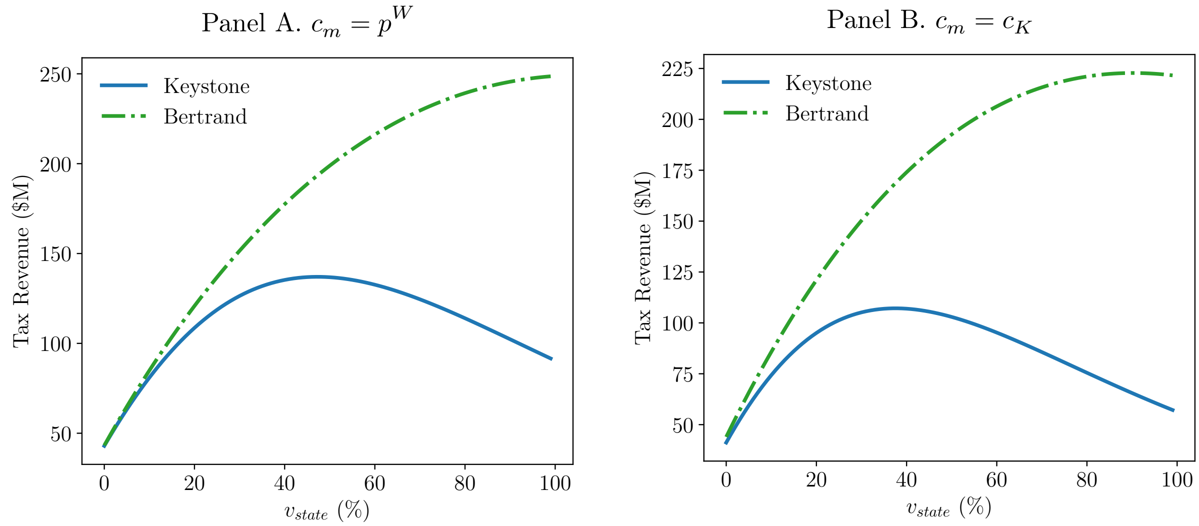
7.4 Counterfactuals

We perform three counterfactual exercises to illustrate the importance of learning conduct for optimal tax policy, focusing on the problem of setting the state-level ad valorem tax for the year 2016. While the government may have other considerations when setting taxes (e.g., curbing consumption to address externalities), we focus solely on revenue raised from the tax. Throughout, we hold market structure and local taxes fixed at their levels in the data.

The first two exercises highlight the importance of learning firm conduct for optimal tax policy counterfactuals. In both, we hold the primitives of demand and cost fixed and only vary the form of conduct. As policymakers might use wholesale price as a proxy for marginal cost, the first exercise sets marginal cost equal to the observed wholesale price. The second exercise imposes the marginal cost implied under Keystone, the model that bests fit the data. In both exercises, we then simulate counterfactual prices, quantities, and government revenue at different levels of the state-wide ad valorem tax for each candidate model.

Our results are plotted in Figure 4. We find that the model of conduct has substantial implications for the Laffer curve when holding demand and cost fixed, in line with the conclusions from the recent literature (Miravete et al., 2018; Hollenbeck and Uetake, 2021; O'Connell and Smith, 2024). For both formulations of cost, the results are similar: the revenue-maximizing state-level tax rate is considerably lower under Keystone than Bertrand. In fact, when using the Keystone-implied costs in Panel B, the revenue maximizing state-level tax rate under Keystone is 38%, almost identical to the 37% rate the government adopted in 2015. The Bertrand model produces very different results; the associated Laffer curve lies well above the Keystone curve and is maximized at $v_{state} = 90\%$ (Panel B).

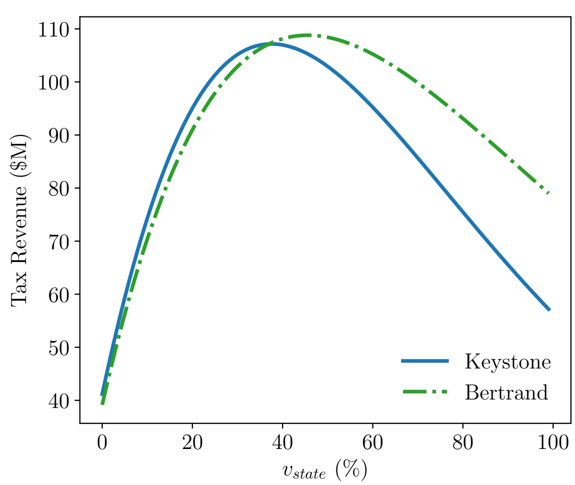
Figure 4: Laffer Curve in 2016 Fixing Cost Across Models



Panels A and B report, for 2016, the Laffer curve implied by different models of conduct for the state-level ad valorem tax (holding local tax rates fixed). Panel A imposes marginal cost equal to the wholesale price. Panel B imposes marginal cost equal to the estimates under Keystone pricing, the preferred model according to our testing results. We then simulate counterfactual prices, quantities, and government revenue at different levels of the tax for each candidate model.

In the third exercise, we consider the thought experiment of a policymaker who commits to a specific model of conduct to both recover costs and simulate the Laffer curve. To compare across assumptions on conduct, we obtain marginal cost estimates under each candidate model. We then use the model-specific cost estimates to simulate counterfactual prices, quantities, and government revenue at different levels of the state-wide ad valorem tax. Our results are plotted in Figure 5.

Figure 5: Laffer Curve in 2016 Under Model Implied Costs



We report for 2016, the Laffer curve implied by different models of conduct for the state-level ad valorem tax (holding local tax rates fixed). Using marginal cost estimates obtained under each model, we simulate counterfactual prices, quantities, and government revenue at different levels of the tax.

Since market outcomes are observed in the data, the Laffer curves for Bertrand and Keystone coincide at the tax rate in the data. Compared to Bertrand, the Keystone Laffer curve is maximized at a lower state-wide ad valorem tax rate (38% vs 46%). Compared to Figure 4, allowing costs to change under each model improves the model's fit and its associated Laffer curve. However, there are quantitatively important consequences to adopting the wrong model. Setting tax at the level implied by Bertrand would lead to a loss of government revenue equal to $2.06 million.

8 Conclusion

We discuss falsification of models of conduct in a general environment where researchers observe market outcomes for firms selling differentiated products. Our results highlight the economic features of different models that permit falsification, including the important role of cost pass-through. Thus, the falsifiable restriction in Berry and Haile (2014) generalizes the pass-through regression used in Sumner (1981) to learn firm conduct.

With our framework, we compare the relevance of various conduct instruments, such as rival cost shifters, product characteristics, and tax rates. In general, we find that different instruments target different features of the pass-through matrix. Furthermore, we show how our framework can guide instrument selection when counterfactual policy prediction is the primary objective. For counterfactuals designed to learn the revenue-maximizing tax, historical variation in the tax rate of interest provides the best instrument for learning conduct.

We demonstrate the usefulness of our framework in a set of simulations and an application to the Washington marijuana market. Appropriately-chosen ad valorem tax instruments conclude that a model of Keystone pricing best fits the data and that the state has chosen virtually the optimal tax rate. We anticipate that our results will offer a useful toolkit for researchers tackling new classes of models or instruments in a wide range of empirical settings.

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Online Appendix

Appendix A Proofs

Proof of Lemma 1. As we note in the text, in our parametric framework, the falsifiable restriction in Equation (28) of Berry and Haile (2014) is[35]

$$E[\omega_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] \quad = \quad E[p_{jt} - \Delta_{mjt} - \bar{c}_{mj}(\mathrm{w}_{jt}) \mid \mathrm{w}_{jt}, z_{jt}] \quad = \quad 0 \qquad a.s.$$

Since observed prices are generated under the true model as

$$p_{jt} \quad = \quad \Delta_{0jt} + c_{0jt} \quad = \quad \Delta_{0jt} + \bar{c}_{0j}(\mathrm{w}_{jt}) + \omega_{0jt}$$

and $E[\omega_{0jt} \mid \mathrm{w}_{jt}, z_{jt}] = 0$ under Assumption 2, the falsifiable restriction is equivalent to

$$E[\Delta_{0jt} + \bar{c}_{0j}(\mathrm{w}_{jt}) + \omega_{0jt} - \Delta_{mjt} - \bar{c}_{mj}(\mathrm{w}_{jt}) \mid \mathrm{w}_{jt}, z_{jt}] \quad = \quad 0 \qquad a.s.$$

or equivalently

$$E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}] \quad = \quad \bar{c}_{mj}(\mathrm{w}_{jt}) - \bar{c}_{0j}(\mathrm{w}_{jt}) \qquad a.s.$$

giving the result. □

Proof of Lemma 2. We prove the inverse of both directions. If the model is not falsified, then there exists a set of cost functions $\{\bar{c}_{mj}(\mathrm{w}_{jt})\}_j$ satisfying the falsifiable restriction. Since neither $\bar{c}_{mj}$ nor $\bar{c}_{0j}$ can depend on the instruments, this means (by Lemma 1) that for each j and each value of w_{jt}, the expectation $E[\Delta_{0jt} - \Delta_{mjt} | \mathrm{w}_{jt}, z_{jt}]$ is almost everywhere constant with respect to z_{jt}. Taking the limit

$$\lim_{h \to 0} \frac{E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt} = \tilde{z}_{jt} + h_k] - E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt} = \tilde{z}_{jt}]}{h}$$

as in the text and noting that this must be 0 almost surely, this becomes

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] = 0 \qquad a.s.$$

giving the result.

[35] See Section 6, Case 2 in Berry and Haile (2014) for a discussion of their non-parametric environment.

For the opposite direction, if for every j and k, $E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] = 0$ almost surely, then $E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}]$ must be the same for almost all values of z_{jt}. If so, define

$$\bar{c}_{mj}(\mathrm{w}_{jt}) \quad = \quad \bar{c}_{0j}(\mathrm{w}_{jt}) + E_{z_{jt}}\left[E[\Delta_{0jt} - \Delta_{mjt} \mid \mathrm{w}_{jt}, z_{jt}]\right]$$

and $\bar{c}_{mj}$ satisfies the equality condition in Lemma 1 almost surely, so the model is not falsified. □

Proof of Proposition 1. See text preceding Proposition 1.

□

Proof of Corollary 1. Let $z_{jt}^{(k)}$, the k-th instrument for product j, be the i^{th} cost shifter of rival product ℓ. Since our instruments are cost shifters, under Assumption 5, $\frac{\partial\Delta_{mjt}}{\partial z_{jt}^{(k)}}$ and $\frac{\partial\Delta_{0jt}}{\partial z_{jt}^{(k)}}$ are both 0. From Proposition 1, then, model m is falsified if for some (j, k),

$$E\left[\left(P_{mt}^{-1} - P_{0t}^{-1}\right)_j \frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p.$$

Since the instrument $z_{jt}^{(k)}$ is a cost shifter of product $\ell \neq j$,

$$\frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} \quad = \quad \frac{\partial p_0}{\partial c_t}\frac{\partial c_t}{\partial z_{jt}^{(k)}} \quad = \quad P_{0t}e_\ell \frac{\partial \bar{c}_{0j}}{\partial \mathrm{w}_{jt}^{(i)}},$$

where e_ℓ is the ℓ-th vector of the canonical basis. As a result, model m is falsified if

$$E\left[\left(P_{mt}^{-1}P_{0t} - I\right)_j e_\ell \frac{\partial \bar{c}_{0j}}{\partial \mathrm{w}_{jt}^{(i)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0 \qquad w.p.p.$$

for some j and some $\ell \neq j$. Since by assumption $\frac{\partial \bar{c}_{0j}}{\partial \mathrm{w}_{jt}^{(i)}} \neq 0$, if we choose ℓ and j such that the (j, ℓ) element of $E[P_{mt}^{-1}P_{0t} \mid \mathrm{w}_{jt}, z_{jt}]$ is nonzero, this condition holds and the model is falsified. □

Proof of Corollary 2. By Lemma 2, falsifiability comes down to whether for some j and k,

$$E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \quad \neq \quad 0 \qquad w.p.p.$$

Let $z_{jt}^{(k)}$ be the i-th characteristic of product ℓ, and let $x_t^{(i)}$ denote the vector of that characteristic for all J products. Note that $x_t^{(i)}$ has both a direct effect on Δ_{mt} and an indirect effect through its impact on equilibrium prices,

$$\frac{\mathrm{d}\Delta_{mt}}{\mathrm{d}x_t^{(i)}} = \frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} + \frac{\partial \Delta_{mt}}{\partial p_t}\frac{\mathrm{d}p_0}{\mathrm{d}x_t^{(i)}}$$

where $\frac{\mathrm{d}p_0}{\mathrm{d}x_t^{(i)}}$ is the effect of $x_t^{(i)}$ on equilibrium prices under the true model 0.

Under Assumption 8, $x_{jt}^{(i)}$ and p_{jt} affect Δ_{mt} and Δ_{0t} *directly* only through δ_{jt}, so

$$\frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} = \frac{\partial \Delta_{mt}}{\partial \delta_t}\frac{\partial \delta_t}{\partial x_t^{(i)}} = \frac{\partial \Delta_{mt}}{\partial \delta_t}\beta^{(i)} I$$

and

$$\frac{\partial \Delta_{mt}}{\partial p_t} = \frac{\partial \Delta_{mt}}{\partial \delta_t}\frac{\partial \delta_t}{\partial p_t} = \frac{\partial \Delta_{mt}}{\partial \delta_t}(-\alpha I)$$

and, putting the two together,

$$\frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} = -\frac{\beta^{(i)}}{\alpha}\frac{\partial \Delta_{mt}}{\partial p_t}$$

We already defined the notation $H_{\Delta_{mt}} = \frac{\partial \Delta_{mt}}{\partial p_t}$, so $\frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} = -\frac{\beta^{(i)}}{\alpha}H_{\Delta_{mt}}$.

Next, to calculate $\frac{\mathrm{d}p_t}{\mathrm{d}x_t^{(i)}}$, recall that equilibrium prices are defined implicitly as the solution to the true first-order conditions $F(\cdot) = p_t - c_t - \Delta_{0t} = 0$. By the implicit function theorem,

$$\frac{\mathrm{d}p_t}{\mathrm{d}x_t^{(i)}} = -\left[\frac{\partial F}{\partial p_t}\right]^{-1}\left[\frac{\partial F}{\partial x_t^{(i)}}\right] = -\left[I - H_{\Delta_{0t}}\right]^{-1}\left[-\frac{\partial \Delta_{0t}}{\partial x_t^{(i)}}\right] = -\left[I - H_{\Delta_{0t}}\right]^{-1}\left[\frac{\beta^{(i)}}{\alpha}H_{\Delta_{0t}}\right]$$

Recalling that $P_{mt} = (I - H_{\Delta_{mt}})^{-1}$, this is

$$\frac{\mathrm{d}p_t}{\mathrm{d}x_t^{(i)}} = -\frac{\beta^{(i)}}{\alpha}P_{0t}\left(I - P_{0t}^{-1}\right) = \frac{\beta^{(i)}}{\alpha}(I - P_{0t})$$

Plugging these into $\frac{\mathrm{d}\Delta_{mt}}{\mathrm{d}x_t^{(i)}} = \frac{\partial \Delta_{mt}}{\partial x_t^{(i)}} + \frac{\partial \Delta_{mt}}{\partial p_t}\frac{\mathrm{d}p_t}{\mathrm{d}x_t^{(i)}}$ gives

$$\frac{\mathrm{d}\Delta_{mt}}{\mathrm{d}x_t^{(i)}} = -\frac{\beta^{(i)}}{\alpha}H_{\Delta_{mt}} + H_{\Delta_{mt}}\left(\frac{\beta^{(i)}}{\alpha}(I - P_{0t})\right) = -\frac{\beta^{(i)}}{\alpha}H_{\Delta_{mt}}P_{0t} = -\frac{\beta^{(i)}}{\alpha}(I - P_{mt}^{-1})P_{0t}$$

From this,

$$
\begin{aligned}
E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] &= E\left[\frac{\beta^{(i)}}{\alpha}\left((P_{0t}^{-1} - P_{mt}^{-1})P_{0t}\right)_{j,\ell} \mid \mathrm{w}_{jt}, z_{jt}\right] \\
&= \frac{\beta^{(i)}}{\alpha} E\left[\left(I - P_{mt}^{-1}P_{0t}\right)_{j,\ell} \mid \mathrm{w}_{jt}, z_{jt}\right]
\end{aligned}
$$

Thus, unless $E[(P_{mt}^{-1}P_{0t})_j \mid \mathrm{w}_{jt}, z_{jt}] = e_j'$ for each j almost surely, there is some (j,k) satisfying $E\left[\frac{\mathrm{d}\Delta_{0jt}}{\mathrm{d}z_{jt}^{(k)}} - \frac{\mathrm{d}\Delta_{mjt}}{\mathrm{d}z_{jt}^{(k)}} \mid \mathrm{w}_{jt}, z_{jt}\right] \neq 0$ w.p.p., the condition for falsifiability under Lemma 2. □

Proof of Proposition 2. Given (3), $c_{0jt} = \bar{c}_{0j}(\mathrm{w}_{jt}) + \omega_{0jt} = \nu_t p_{jt} - \tau_t - \nu_t \Delta_{0jt}$ and $c_{mjt} = \bar{c}_{mj}(\mathrm{w}_{jt}) + \omega_{mjt} = \nu_t p_{jt} - \tau_t - \nu_t \Delta_{mjt}$, so

$$
\begin{aligned}
\omega_{mjt} &= \nu_t p_{jt} - \tau_t - \nu_t \Delta_{mjt} - \bar{c}_{mj}(\mathrm{w}_{jt}) + \bar{c}_{0j}(\mathrm{w}_{jt}) + \omega_{0jt} - \nu_t p_{jt} + \tau_t + \nu_t \Delta_{0jt} \\
&= \nu_t \Delta_{0jt} - \nu_t \Delta_{mjt} + \bar{c}_{0j}(\mathrm{w}_{jt}) - \bar{c}_{mj}(\mathrm{w}_{jt}) + \omega_{0jt}
\end{aligned}
$$

Since $\bar{c}_{0j}$, $\bar{c}_{mj}$, and $E(\omega_{0jt}|\mathrm{w}_{jt}, z_{jt})$ don't depend on the instruments, falsification occurs if $E(\nu_t\Delta_{0jt} - \nu_t\Delta_{mjt}|\mathrm{w}_{jt}, z_{jt})$ for some j varies with one of the instruments. Since

$$
\frac{\mathrm{d}(\nu_t\Delta_{0jt})}{\mathrm{d}z_{jt}^{(k)}} = \nu_t \frac{\partial \Delta_{0jt}}{\partial z_{jt}^{(k)}} + \nu_t \frac{\partial \Delta_{0jt}}{\partial p_t}\frac{\mathrm{d}p_0}{\mathrm{d}z_{jt}^{(k)}} + \Delta_{0jt}\frac{\mathrm{d}\nu_t}{\mathrm{d}z_{jt}^{(k)}}
$$

and $\frac{\partial \Delta_{0jt}}{\partial p_t} = \frac{\partial(p_t - c_{0t})}{\partial p_t} = I - P_{0t}^{-1}$, and the analogous expression for $\frac{\mathrm{d}(\nu_t\Delta_{mjt})}{\mathrm{d}z_{jt}^{(k)}}$, (4) follows. □

Proof of Corollary 3. Since the instrument is $z_t = \tau_t$, $\frac{\mathrm{d}\nu_t}{\mathrm{d}z_{jt}^{(k)}} = 0$; and under Assumption 9, the tax rate doesn't enter into Δ_{mjt} or Δ_{0jt} directly, so falsification obtains if

$$
E\left[\nu_t(P_{mt}^{-1} - P_{0t}^{-1})_j \frac{\mathrm{d}p_0}{\mathrm{d}\tau_t}\middle| \mathrm{w}_{jt}, \tau_t\right] \neq 0
$$

with positive probability for some j. The unit tax is the same as an increase in marginal costs for every product, so $\frac{\mathrm{d}p_0}{\mathrm{d}\tau_t} = \sum_j \frac{\mathrm{d}p_0}{\mathrm{d}c_{jt}} = P_0\iota$, where ι is a vector of ones. Falsification therefore requires that for some j, with positive probability over observables,

$$
\begin{aligned}
E\left[\nu_t(P_{mt}^{-1} - P_{0t}^{-1})_j P_{0t}\iota\middle| \mathrm{w}_{jt}, \tau_t\right] &\neq 0 \\
&\updownarrow \\
E\left[\nu_t(P_{mt}^{-1}P_{0t} - I)_j\iota\middle| \mathrm{w}_{jt}, \tau_t\right] &\neq 0
\end{aligned}
$$

or the elements of the j^{th} row of $P_{mt}^{-1}P_{0t}$ don't sum to 1. $\square$

Proof of Corollary 4. With the added assumption that either $P_{mt}^{-1} = P_{0t}^{-1}$ or $\frac{\mathrm{d}p_0}{\mathrm{d}z_t} = 0$, the first term in Equation 4 vanishes; so with $z_t = \nu_t$, under Proposition 2, falsification obtains if and only if for some j, $E\left[\Delta_{0jt} - \Delta_{mjt}\middle|\, \mathrm{w}_{jt}, \nu_t\right] \neq 0$ w.p.p., giving the result. $\square$

Proof of Corollary 5. With ν_t fixed, government revenue at unit tax rate τ is $R_0(\tau) = \tau \sum_j s_{jt}(p_{0t}(\tau)) + (1 - \nu_t) \sum_j p_{0t}(\tau) s_{jt}(p_{0t}(\tau))$; let $R_m(\tau)$ be the predicted revenue under model m, based on predicted prices $p_{mt}(\tau)$. As noted in the text, $\frac{dp_{mt}}{d\tau} = \frac{1}{\nu_t} P_{mt}\iota$ for either model. If $P_{mt}\iota = P_{0t}\iota$ everywhere and model m and the truth give the same prices at the observed tax rate $\tau = \tau_t$, then they predict the same prices $p_{mt}(\tau) = p_{0t}(\tau)$ at every tax rate, and therefore the same market shares $s_{jt}(p_{mt}(\tau)) = s_{jt}(p_{0t}(\tau))$ and revenue $R_m(\tau) = R_0(\tau)$. $\square$

Proof of Corollary 6. With τ_t fixed, government revenue under ad valorem tax rate ν is $R_0(\nu) = \tau_t \sum_j s_{jt}(p_{0t}(\nu)) + (1 - \nu) \sum_j p_{0t}(\nu) s_{jt}(p_{0t}(\nu))$; let $R_m(\nu)$ be the predicted revenue under model m. As noted in the text, for either model, $\frac{\mathrm{d}p_{mt}}{d\nu} = \frac{1}{\nu} P_{mt}(\Delta_{mt} - p_{mt})$. If $P_{mt}(\Delta_{mt} - p_{mt}) = P_{0t}(\Delta_{0t} - p_{0t})$ whenever $p_{mt} = p_{0t}$ and $p_{mt} = p_{0t}$ at the observed tax rate $\nu = \nu_t$, then $p_{mt}(\nu) = p_{0t}(\nu)$ for all tax rates, and the two models therefore predict the same market shares and revenue at every tax rate. $\square$

Appendix B Connecting Falsification and Point Identification when Models are Nested

The results in Sections 3 and 4 are cast in terms of falsification of a candidate model m, and we pursue a testing approach in our simulations and application, but the results in the paper are also useful to inform identification (and thus estimation) exercises. In particular, our results can guide ex-ante instrument selection to avoid irrelevant instruments for estimation.

Similar to Magnolfi and Sullivan (2022), consider a setting where different candidate models belong to a parametric class, so that markups can be written as $\Delta(\theta)$ for a vector of parameters $\theta \in \Theta$. In this class of models, falsification proceeds as in our general discussion in Sections 3 and 4. In particular, a model m corresponding to a parameter value θ_m, is falsified by instruments z_{jt} under the general conditions laid out in Proposition 1. Within the parametric class of nested models, however, we can also discuss the identification of the true model, characterized by θ_0. For any value of θ, let the implied cost be $p_{jt} - \Delta_{jt}(\theta) = c(\mathrm{w}_{jt}; \theta) = \bar{c}(\mathrm{w}_{jt}; \theta) + \omega_{jt}(\theta)$. We say that the true model is *point identified by the instruments* z_{jt} when $E[\omega_{jt}(\theta) | \mathrm{w}_{jt}, z_{jt}] = 0$ if and only if $\theta = \theta_0$. Thus, point identification requires that all models m for which $\theta_m \neq \theta_0$ are falsified by the instruments z_{jt}. Conversely, when there exist a a model m such that $\theta_m \neq \theta_0$ which is not falsified by z_{jt}, θ_0 is not point identified by these instruments. Clearly, instruments that ensure falsification for all incorrect models in the class are prime candidates to be used for identification and estimation. However, instruments for which an incorrect model is not falsified would lead to a failure of point identification.

As an example of a conduct model defined by a continuous parameter, IO economists often use profit weights as a reduced-form way to model collusion or common ownership (e.g., Backus et al. (2021)). We illustrate how our framework for understanding falsification through pass-through can be useful for ex-ante instrument selection for identification and estimation with two examples based on such profit weights.

Example 8: We remain in the example environment of the paper – two single-product firms and logit demand. Suppose the two firms compete in quantities a la Cournot, but instead of maximizing its own profit, each firm maximizes a weighted sum of its own and the other firm's profits. Specifically, each firm j chooses quantity s_{jt} to solve

$$\max_{s_{jt}} \left\{ s_{jt}(p_{jt}(\cdot) - c_{jt}) + \theta_j s_{-jt}(p_{-jt}(\cdot) - c_{-jt}) \right\}$$

where $-j$ refers to the identity of the rival firm. This nests standard Cournot competition (when $\theta_1 = \theta_2 = 0$) and perfect collusion/joint profit maximization (when $\theta_1 = \theta_2 = 1$), along with intermediate cases that might be interpreted as "imperfect collusion". The corresponding markups under logit demand are

$$\Delta_{WCt}(\theta) = \begin{bmatrix} \frac{1-(1-\theta_1)s_{2t}}{\alpha s_{0t}} \\ \frac{1-(1-\theta_2)s_{1t}}{\alpha s_{0t}} \end{bmatrix},$$

with WC standing for the Weighted Cournot model.

For this setting, our falsification framework can be used to show that either cost or demand side instruments allow point identification of the profit weights $\theta = (\theta_1, \theta_2)$:

Result 1. *If the data are generated by Cournot competition with profit weight equal to $\theta = \theta_0 \in [0,1]^2$, then either cost side or demand side instruments point identify the true parameter.*

To see this result, we first simplify the first-order conditions and work out the pass-through and inverse pass-through matrices

$$P_{WCt}^{-1} = \frac{1}{s_{0t}} \begin{bmatrix} 1 - s_{2t} & \theta_1 s_{2t} \\ \theta_2 s_{1t} & 1 - s_{1t} \end{bmatrix} \qquad \text{and} \qquad P_{WCt} = \frac{s_{0t}}{\kappa_{WCt}} \begin{bmatrix} 1 - s_{1t} & -\theta_1 s_{2t} \\ -\theta_2 s_{1t} & 1 - s_{2t} \end{bmatrix}$$

where $\kappa_{WCt} = (1 - s_{1t})(1 - s_{2t}) - \theta_1\theta_2 s_{1t}s_{2t}$.

Suppose the true model is weighted Cournot competition with weights $\theta_0 = (\theta_{01}, \theta_{02})$, and we are interested in falsifying a model of weighted Cournot with misspecified weights $\theta_m = (\theta_{m1}, \theta_{m2}) \neq \theta_0$. Focusing on the off-diagonal terms and dropping constants, we can calculate

$$P_{WCmt}^{-1} P_{WC0t} \quad \propto \quad \begin{bmatrix} \star & (\theta_{m1} - \theta_{01}) s_{2t}(1 - s_{2t}) \\ (\theta_{m2} - \theta_{02}) s_{1t}(1 - s_{1t}) & \star \end{bmatrix}$$

Thus, if $\theta_{m1} > \theta_{01}$, the top-right off-diagonal is always positive, and therefore positive when its expectation is taken over unobservables; if $\theta_{m1} < \theta_{01}$, it's always negative, hence negative in expectation. Likewise, if $\theta_{m2} > \theta_{02}$, the bottom-right term is always positive, and if $\theta_{m2} < \theta_{02}$ always negative. Thus, if $\theta_m \neq \theta_0$, the matrix $[P_m^{-1} P_0]^*$ is not diagonal, so under Corollary 1, any incorrect model within the class is falsified by cost side instruments under Corollary 1. As for demand side instruments, the profit-weighted Cournot model satisfies Assumption 7 and logit demand satisfies Assumption 8, so since $[P_m^{-1} P_0]^*$ is not the identity matrix, demand side instruments falsify any incorrect model under Corollary 2. As a result, either type of instrument allows for point identification of θ_0. •

Example 9: In the same environment, suppose now that the two firms compete in prices with profit weights. For simplicity, suppose the two firms' profit weights are the same, so each firm j chooses price p_{jt} to solve

$$\max_{p_{jt}} \left\{ (p_{jt} - c_{jt}) s_{jt}(p_t) + \theta (p_{-jt} - c_{-jt}) s_{-jt}(p_t) \right\}$$

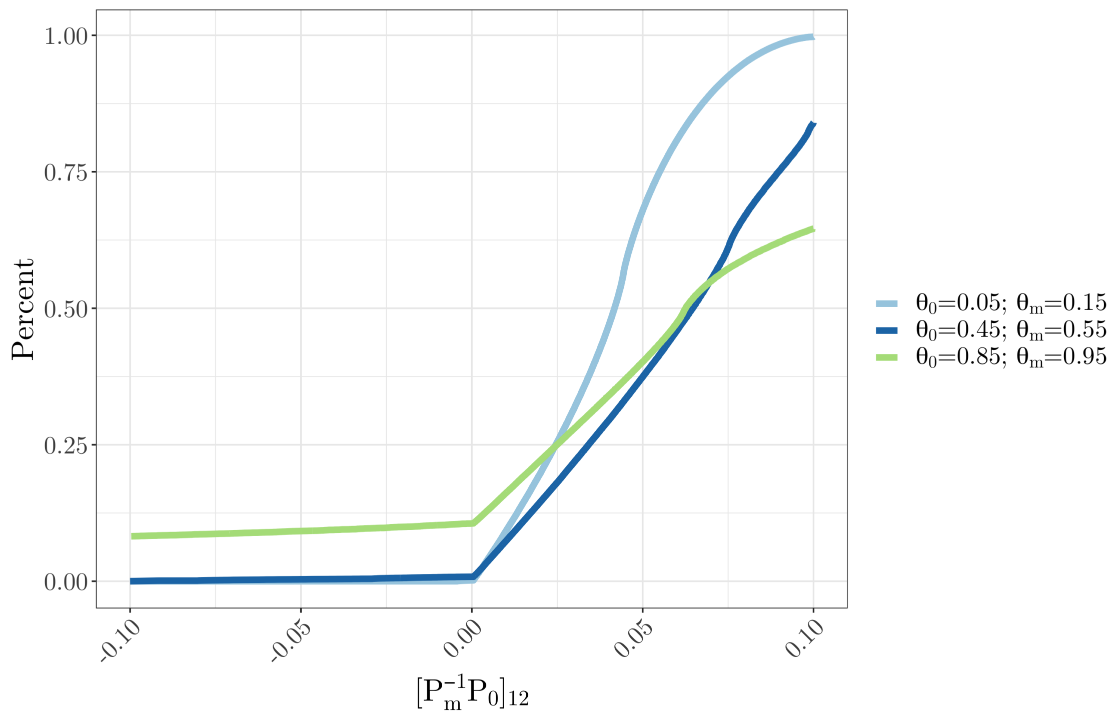
Under logit demand, the two firms' first-order conditions give the markup function

$$\Delta_{WBt}(\theta) \quad = \quad \begin{bmatrix} \frac{1-(1-\theta)s_{2t}}{\alpha s_{0t} + \alpha(1-\theta^2)s_{1t}s_{2t}} \\ \\ \frac{1-(1-\theta)s_{1t}}{\alpha s_{0t} + \alpha(1-\theta^2)s_{1t}s_{2t}} \end{bmatrix}$$

with WB standing for the Weighted Bertrand model.

While we obtain identification in Weighted Cournot, irrespective of the value of θ_0 and of other observables, in the case of Weighted Bertrand, results are more nuanced. For a given

Figure 6: CDF of $[P_{mt}^{-1} P_{0t}]_{12}$ for Betrand Profit Weight Models



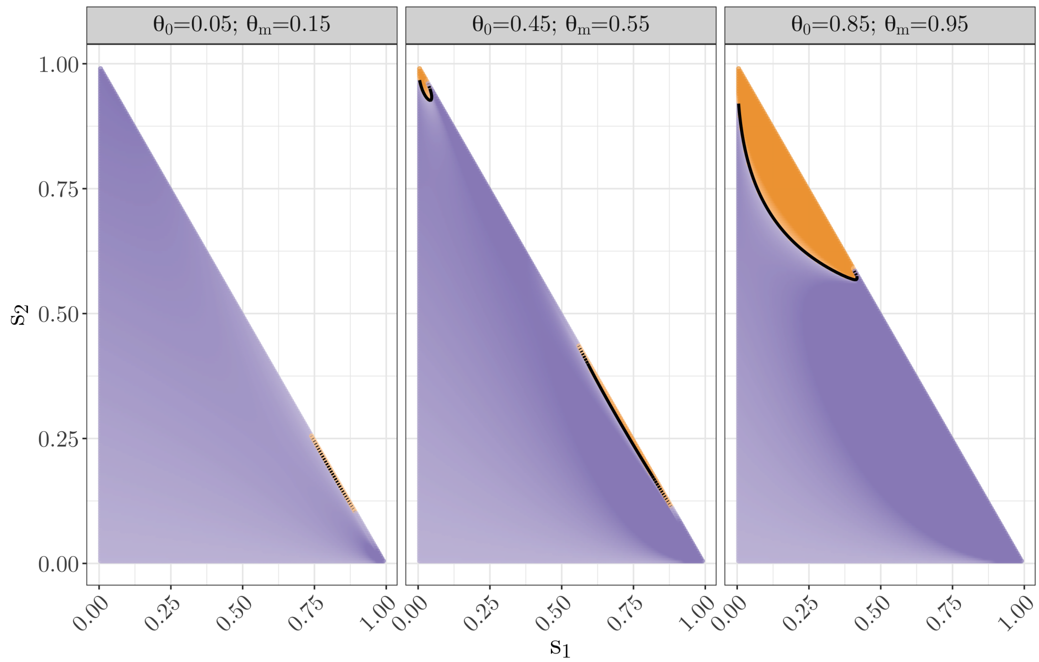
This figure plots distributions of $[P_{mt}^{-1} P_{0t}]_{12}$ over realizations of (s_{1t}, s_{2t}) for three different values of (θ_0, θ_m).

true value of θ_0 and a given misspecified model $\theta_m \neq \theta_0$, the off-diagonal terms of the matrix $P_m^{-1} P_0$ can be either positive or negative, but are not typically zero; and the diagonal terms can be greater than or less than 1, but are not typically equal to either 1 or to each other. This means there are realizations of observables (w_t, z_t) for which $[P_m^{-1} P_0]^\star$ could in principle be diagonal, but it won't typically be – for example, because some off-diagonal element of $P_{mt}^{-1} P_{0t}$ varies continuously and could take either sign, so its expectation being exactly zero requires a non-generic distribution of unobservables. Still, falsification could in principle fail, and therefore identification could fail as well.

To illustrate this possibility, we compute the top-right off-diagonal term, $[P_m^{-1} P_0]_{12}$, for a large sample of uniform draws of (s_{1t}, s_{2t}), and display the resulting distribution in Figure 6 for three representative pairs of true and misspecified models (θ_0, θ_m). While the distributions all have support that includes 0, the distributions are continuous, so getting a conditional expectation $E([P_{mt}^{-1} P_{0t}]_{12}) = 0$ would only hold for a non-generic set of observables.

We can also examine the value of the off-diagonal term at each point in the simplex of realizations of market shares (s_{1t}, s_{2t}). In each panel of Figure 7, we do so for three pairs of θ_0 and θ_m such that $\theta_m - \theta_0 = 0.1$. For each realization on the simplex, we indicate the value of $[P_m^{-1} P_0]_{12}$ with color gradients – darker orange indicates negative values that are larger in magnitude and darker purple indicates positive values that are larger in magnitude.

FIGURE 7: Values of $[P_{mt}^{-1}P_{0t}]_{12}$ for Realizations of Market Shares in Bertrand Profit Weight Models



This figure plots magnitudes of $[P_{mt}^{-1}P_{0t}]_{12}$ over the simplex of (s_{1t}, s_{2t}) for three different values of (θ_0, θ_m). Darker purple (orange) shading indicates more positive (negative) values. Black indicates a zero value.

The points where $[P_m^{-1}P_0]_{12} = 0$ are indicated in black. Immediately, one sees that for the vast majority of realizations of shares, $[P_m^{-1}P_0]_{12} > 0$. In fact, $[P_m^{-1}P_0]_{12}$ is always positive whenever the market share of both products is below 0.6. For an empirically relevant example, take the setting in Miller and Weinberg (2017). There, the outside option is defined in such a way that the market share of any product is less than 0.5 in all markets. In that case, the average of $[P_m^{-1}P_0]_{12}$ is positive in all three panels, and falsification of the wrong profit weight is possible with cost or demand side instruments.

As in the previous example, the conduct models satisfy Assumption 7 and the demand system satisfies Assumption 8, so $[P_m^{-1}P_0]^*$ being non-diagonal suffices for falsification of the wrong model. Thus, for either cost or demand side instruments, point identification is not theoretically guaranteed for a particular realization of observables; but with variation in observables, seems virtually guaranteed, as the knife-edge result of positive and negative values of $[P_m^{-1}P_0]_{12}$ cancelling out in expectation seems impossible across different realizations of observables. •

Appendix C Markup Assumption

Assumption 7 holds naturally for a wide range of models where firms choose actions to maximize profits. We suppress the market index t and suppose for simplicity that products $i = 1$ through f are sold by the same firm, and that the firm chooses a set of actions $\{a_i\}_{i=1}^{f}$ to maximize profits,

$$\max_{\{a_i\}, i \in f} \sum_{i \in f} (p_i(a) - c_i) s_i(a)$$

where prices $p(\cdot)$ and market shares $s(\cdot)$ are determined by the actions taken by all firms. First-order conditions are then

$$\sum_{i=1}^{f} \frac{\partial p_i}{\partial a_j} s_i + \sum_{i=1}^{f} (p_i - c_i) \frac{\partial s_i}{\partial a_j} \quad = \quad 0$$

or

$$\begin{bmatrix} \frac{\partial s_1}{\partial a_1} & \frac{\partial s_2}{\partial a_1} & \cdots & \frac{\partial s_f}{\partial a_1} \\ \frac{\partial s_1}{\partial a_2} & \frac{\partial s_2}{\partial a_2} & \cdots & \frac{\partial s_f}{\partial a_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial s_1}{\partial a_f} & \frac{\partial s_2}{\partial a_f} & \cdots & \frac{\partial s_f}{\partial a_f} \end{bmatrix} \begin{bmatrix} p_1 - c_1 \\ p_2 - c_2 \\ \vdots \\ p_f - c_f \end{bmatrix} = - \begin{bmatrix} \sum_{i \in f} s_i \frac{\partial p_i}{\partial a_1} \\ \sum_{i \in f} s_i \frac{\partial p_i}{\partial a_2} \\ \vdots \\ \sum_{i \in f} s_i \frac{\partial p_i}{\partial a_f} \end{bmatrix}$$

Stacking across firms, we then get

$$\left[\Omega \odot \left[\frac{\partial s}{\partial a} \right]' \right] \Delta \quad = \quad - \left[\Omega \odot \left[\frac{\partial p}{\partial a} \right]' \right] s$$

where Ω is the ownership matrix,[36] and therefore

$$\Delta \quad = \quad - \left[\Omega \odot \left[\frac{\partial s}{\partial a} \right]' \right]^{-1} \left[\Omega \odot \left[\frac{\partial p}{\partial a} \right]' \right] s$$

Note that the right-hand side has no room for costs or product characteristics to enter directly – it's all just ownership structure and the way that firm actions a map to market outcomes (p, s), which depends on the demand system.

(Within this more general model, Bertrand is just the special case where firms choose

[36]This is defined as $\Omega_{ij} = 1$ if products i and j are sold by the same firm, and zero otherwise.

prices, so $p(a) = a$ and therefore $\frac{\partial p}{\partial a} = I$; and Cournot is the special case where firms choose quantities so $s(a) = a$ and $\frac{\partial s}{\partial a} = I$. Here we're being more general about what exactly firms are choosing, and therefore what exactly they're assuming other firms are holding fixed while they optimize.)

This assumption also holds if firms maximize any weighted sum of their own profits, other firms' profits, and consumer surplus (or total welfare). Suppose the firm selling product j maximizes

$$\sum_{i=1}^{J} \gamma_{ji}(p_i - c_i)s_i + \lambda_j CS$$

where γ_{ji} is the weight the firm puts on the profits from product i (whether or not i is one of the same firm's products) and CS is consumer surplus. The first-order condition with respect to action a_j is then

$$\sum_{i=1}^{J} \gamma_{ji} \frac{\partial p_i}{\partial a_j} s_i + \sum_{i=1}^{J} \gamma_{ji}(p_i - c_i) \frac{\partial s_i}{\partial a_j} - \lambda_j s_j \quad = \quad 0$$

or, stacking and rearranging,

$$\Delta \quad = \quad \left[\Gamma \odot \left[\frac{\partial s}{\partial a} \right]' \right]^{-1} \left[\Lambda - \Gamma \odot \left[\frac{\partial p}{\partial a} \right]' \right] s$$

where Γ is a matrix of the γ_{ji} terms and Λ is a diagonal matrix of the λ_j terms. Once again, the right-hand side contains only constants and features of the demand system, not costs or product characteristics.

Finally, consider a market with some first-movers and some second-movers. To avoid getting bogged down in notation, we show the result for two single-product firms facing general demand, but the intuition is the same more generally. Conditional on the action a_1 chosen by the first-mover, the second-mover chooses a_2 to maximize $(p_2 - c_2)s_2$, giving first-order condition

$$(p_2 - c_2) \frac{\partial s_2}{\partial a_2} + \frac{\partial p_2}{\partial a_2} s_2 \quad = \quad 0$$

Defining $F(a_1, a_2)$ as the left-hand side, then, a_2 is implicitly defined as a function of a_1 as the solution to $F(a_1, a_2) = 0$, so by the implicit function theorem,

$$a_2'(a_1) \quad = \quad -\frac{\frac{\partial F}{\partial a_1}}{\frac{\partial F}{\partial a_2}} \quad = \quad -\frac{\frac{\partial p_2}{\partial a_1}\frac{\partial s_2}{\partial a_2} + (p_2 - c_2)\frac{\partial^2 s_2}{\partial a_2 \partial a_1} + \frac{\partial^2 p_2}{\partial a_1 \partial a_2} s_2 + \frac{\partial p_2}{\partial a_2}\frac{\partial s_2}{\partial a_1}}{\frac{\partial p_2}{\partial a_2}\frac{\partial s_2}{\partial a_2} + (p_2 - c_2)\frac{\partial^2 s_2}{\partial a_2^2} + \frac{\partial^2 p_2}{\partial a_2^2} s_2 + \frac{\partial p_2}{\partial a_2}\frac{\partial s_2}{\partial a_2}}$$

We can go a step further, rewriting firm 2's first-order condition as $p_2 - c_2 = -\frac{\partial p_2}{\partial a_2} s_2 \Big/ \frac{\partial s_2}{\partial a_2}$ and plugging that into the expression for a_2', to emphasize that a_2 depends only on features of the demand system (how p and s respond to a) and therefore not directly on costs. The first-mover's problem is

$$\max(p_1(a_1, a_2(a_1)) - c_1)s_1(a_1, a_2(a_1))$$

with first-order condition

$$\frac{\partial p_1}{\partial a_1} s_1 + \frac{\partial p_1}{\partial a_2} a_2' s_1 + (p_1 - c_1)\frac{\partial s_1}{\partial a_1} + (p_1 - c_1)\frac{\partial s_1}{\partial a_2} a_2' \quad = \quad 0$$

whence

$$p_1 - c_1 \quad = \quad -\frac{\frac{\partial p_1}{\partial a_1} s_1 + \frac{\partial p_1}{\partial a_2} a_2' s_1}{\frac{\partial s_1}{\partial a_1} + \frac{\partial s_1}{\partial a_2} a_2'}$$

We therefore have both firms' markups $p_j - c_j$ as functions of the demand system, with no place for marginal costs or product characteristics to enter directly. If we had infinite patience, we could make this same argument for the general model of many multi-product firms with some first- and some second-movers, and by induction, with more than two "rounds" of actions.

Appendix D Simulation Details

We provide further details on the simulation environment used in Section 6. Using the simulation class in PyBLP (Conlon and Gortmaker (2020)), we simulate data for 50,000 markets. In each market t, the number of products J_t is a randomly chosen integer between two to ten, leaving us with 319,719 observations in the sample. Each product j is produced by a single product firm.

For demand, we adopt a simple logit framework, in line with our falsification examples. Consumer i gets indirect utility of consuming product j in market t given by:

$$u_{ijt} = x_{jt}\beta + \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

where x_{jt} is a vector containing a constant and two observed product characteristics (x_{1jt} and x_{2jt}), p_{jt} is the price of the product, and ξ_{jt} and ϵ_{ijt} are unobservable shocks at the product-market and the individual product market level, respectively. The utility of the outside option is normalized to $u_{i0t} = \epsilon_{i0t}$. We draw each observed product characteristic x_{1jt} and x_{2jt} independently from the uniform distribution $U(0, 3)$, while ϵ_{ijt} is assumed to be distributed Type I extreme value. The distribution of the unobserved demand shocks ξ_{jt} is discussed below. The mean taste parameters β are set as $\beta = [1, 2, 1]$ while the price parameter $\alpha = -0.5$.

On the supply side, we assume that the marginal cost of producing product j in market t is $c_{jt} = \mathrm{w}_{jt}\gamma + \omega_{0jt}$ where where w_{jt} is a vector containing a constant and two observed cost shifters (w_{1jt} and w_{2jt}) which are excluded from demand. Marginal cost also depends on ω_{0jt} the true unobserved cost shock. As with the product characteristics, we draw each observed cost shifter w_{1jt} and w_{2jt} independently from the uniform distribution $U(0, 3)$. We adopt the default in PyBLP by drawing the unobserved demand and cost shocks ξ_{jt} and ω_{0jt} from a mean zero bivariate normal distribution with variances of 1 and a correlation of 0.9. We set $\gamma = [3, 0.5, 1.5]$. For simplicity, the market size is normalized to one for all t. The government levies both a unit tax (τ_t) and an ad valorem tax (υ_t) on all products in market t. The unit tax is remitted by the firms while consumers remit the ad valorem tax. We draw the unit tax in each market from the uniform distribution $U(1, 2)$ while the ad valorem tax in each market is drawn from the uniform distribution $U(0, 0.2)$. We assume that the true model of conduct is Keystone pricing, whereby firms set tax exclusive prices as twice their marginal cost, or $\nu_t p_{jt} = 2c_{jt} + 2\tau_t$.

Appendix E Summary of IV Relevance in Examples

To help the reader, we now summarize the takeaways from the examples in the paper. Recall that we consider a stylized environment with two single-product firms, logit demand, and no unobservable variation in demand and cost. In the following table, for select combinations of true and alternative model, we indicate which of the sources of variation considered in the examples will permit falsification (✓), and which ones will not (×).

True	Tested	Cost Side	Demand Side	Unit Tax	Ad Valorem Tax
Bertrand	Keystone	✓	✓	✓	✓
	Cournot	✓	✓	✓	✓
	MC Pricing	✓	✓	✓	✓
Cournot	Keystone	×	✓	✓	✓
	MC Pricing	×	✓	✓	✓
Keystone	MC Pricing	×	×	✓	×
$\Delta_{0t} = \zeta_{0t}$	$\Delta_{mt} = \zeta_{mt}$	×	×	×	✓

Appendix F Additional Details: Data and Demand

F.1 Data Cleaning and Description

Table 4 summarizes the main data cleaning steps for our transaction-level data. We then take the cleaned transaction-level data and aggregate to the product-market level.

Table 4: Data Sample Cleaning Steps

		Resulting	
Step	**Sample Restriction**	**Sample Size**	**Revenue ($)**
0	Original	74,427,564	1,213,615,962
1	Keep usable products	53,883,838	813,353,991
2	Drop package size $\neq$ inventory usable weight	53,791,104	809,548,311
3	Keep 1 and 3.5 g package sizes	34,418,816	490,387,068
4	Drop first 15 days of establishment's sales	34,286,246	488,555,343
5	Keep $\frac{\text{retail prices}}{\text{wholesale prices}} \in [1, 5]$	34,170,796	486,487,909
6	Keep weight $\leq$ 10 g	34,139,758	482,017,967

Table reports all steps to clean transaction-level data and their effect on sample size.

Table 5 provides descriptive statistics for the main variables in our database.

Table 5: Summary Statistics

	Mean	SD	Min	P25	Median	P75	Max
Price (per gram, tax inclusive)	11.04	3.05	4.54	9.00	10.89	12.86	21.07
Price (per gram, tax exclusive)	7.61	2.18	3.11	6.18	7.48	8.85	15.60
Shares (percent)	0.38	0.90	0.00	0.02	0.08	0.34	39.06
Wholesale Price (per gram)	3.60	0.91	1.50	3.00	3.58	4.09	6.50
Size	2.19	1.25	1.00	1.00	1.00	3.50	3.50
THC	5.37	7.90	0.00	0.53	0.99	6.80	62.66
CBD	0.42	1.56	0.00	0.00	0.09	0.22	36.93
Rival Products	443.67	571.17	1.00	60.00	196.00	499.00	2109.00
Firms by Market	13.80	15.55	2.00	3.00	7.00	16.00	51.00
Processors by Market	111.09	76.46	1.00	49.00	91.00	149.00	316.00
Local Tax Rate	0.089	0.007	0.070	0.085	0.088	0.095	0.103
Observations	153,936						

This table reports summary statistics for the main variables in our database.

F.2 Demand Estimation

We provide here further details on the demand system introduced in Section 7.2. In our demand system, each consumer i receives utility from product j in market t according to the indirect utility:

$$u_{ijt} = x_{jt}\beta_i + \alpha_i p_{jt} + F_{r(j)} + F_{\ell(j)} + F_{m(t)} + \xi_{jt} + \zeta_{it} + (1-\rho)\epsilon_{ijt}$$

where x_j includes a constant, package size, THC and CBD (and their values squared), and the log of the number of products offered in the store; we include this variable to capture variation in shelf space across stores. The variable p_{jt} is the price of product j in market t, and $F_{r(j)}$, $F_{\ell(j)}$, $F_{m(t)}$ denote fixed effects for the retailer selling product j, the processor producing product j, and the year-month of the retail transaction respectively. Consumer preferences for characteristics ($\beta_i = \bar{\beta} + \tilde{\beta} \times \text{Income}_i$) and price ($\alpha_i = \bar{\alpha} + \tilde{\alpha} \times \text{Income}_i$) vary with individual level income. ξ_{jt} and $\zeta_{it} + (1-\rho)\epsilon_{ijt}$ are unobservable shocks at the product-market and the individual product market level, respectively. Following the nested logit structure for our choice of nesting all inside goods together, ϵ_{ijt} is distributed Type 1 Extreme Value, and ζ_{it} is distributed according to the conjugate distribution (Cardell (1997)). To close the model we normalize consumer i's utility from the outside option as $u_{i0t} = \epsilon_{i0t}$. Given this utility specification, market shares s_{jt} as a function of observables, unobservables and parameters take on the standard form (Berry, Levinsohn, and Pakes (1995); Grigolon and Verboven (2014)).

Identification and Estimation: The identifying assumption for our demand model is that, for a vector of demand instruments z^d_{jt}, the moment condition $E[\xi_{jt} z^d_{jt}] = 0$. We construct several demand instruments. We first construct the number of products sold at competing dispensaries in market t to help identify ρ. Following Gandhi and Houde (2023) we also interact this instrument with the mean income in the market to help identify income interaction parameters. We further construct BLP-style instruments to capture the closeness of products in the product space. Specifically, in a market, we sum the amount of THC and CBD both for products within a given store and also for products in all other stores. We also include exogenous own cost shifters including the amount of rainfall and temperature in the region of production and their lags.

Because we do not take a stance on conduct, we perform demand estimation without using any supply-side moments.[37] However, we specify in Section 7.3 a menu of candidate models of conduct which includes Keystone pricing, Bertrand, and marginal cost pricing.

[37] See Appendix I of Duarte et al. (2024) for a comparison of sequential and simultaneous approach to conduct testing.

The set of instruments specified above includes shifters of prices (e.g., own cost shifters) that will be relevant under any conduct model in our menu including Keystone and marginal cost pricing.

Table 6: Demand Estimates

	(1) Logit-OLS		(2) Logit-2SLS		(3) RCNL	
	coef	s.e.	coef	s.e	coef	s.e
Price (in \$)	-0.102	(0.002)	-0.283	(0.042)	-0.455	(0.066)
Package Size (= 3.5 oz)	0.441	(0.007)	0.335	(0.025)	0.110	(0.048)
THC	0.077	(0.003)	0.062	(0.005)	0.027	(0.007)
THC Squared	-0.003	(0.000)	-0.003	(0.000)	-0.001	(0.000)
CBD	-0.095	(0.009)	-0.084	(0.009)	-0.046	(0.010)
CBD Squared	-0.002	(0.001)	-0.002	(0.001)	-0.002	(0.000)
Log Number Own Products	-0.021	(0.016)	0.001	(0.017)	0.115	(0.027)
Constant	-5.791	(0.061)				
ρ					0.282	(0.059)
Income × Constant					-0.005	(0.003)
Income × Price					0.004	(0.002)
Median Own Price Elasticity	-1.113		-3.079		-6.451	
Median Aggregate Price Elasticity	-0.796		-2.192		-3.324	
Diversion to outside option	0.603		0.603		0.434	
Retailer FE	Yes		Yes		Yes	
Processor FE	Yes		Yes		Yes	
Year-Month FE	Yes		Yes		Yes	

Demand estimates for a logit model of demand obtained from OLS estimation are reported in column 1 and 2SLS estimation are reported in column 2. Column 3 reports estimates for the full RCNL demand model. Income is measured in \$100,000. $n = 153,936$.

Results: Results for demand estimation are reported in Table 6. For reference, we also report, in Columns 1 and 2, estimates from a simple logit model. In Column 1, we estimate the model with ordinary least squares. In Column 2, we estimate the model via 2SLS, using the same instruments as in the main specification. The reduction in the price coefficient between Columns 1 and 2, indicates the presence of endogeneity not controlled for by the fixed effects. Column 3 reports estimates of the full demand model. Compared to columns

1 and 2, the full model yields a more elastic demand system with diversion to the outside option that departs from logit.

Appendix G Robustness Checks

We present here additional robustness results for our empirical application.

G.1 Robustness to Box-Cox Transformations of Income

Following Miravete et al. (2024) we incorporate in our demand system a more flexible specification of income effects by adopting a Box-Cox transformation. Specifically, we allow consumer i's price sensitivity parameter α_i to depend on a nonlinear transformation of income (y_i), so that

$$\alpha_i = \bar{\alpha} + \tilde{\alpha} \times y_i^{\lambda-1}. \tag{6}$$

Our main specification corresponds to $\lambda = 2$. In principle, λ could be estimated as an additional parameter of the demand system. Because we lack the variation to credibly identify λ in our empirical environment, here we consider the robustness of our testing results to re-estimating our demand system for calibrated values of $\lambda \in \{0, 0.5, 1.5\}$. Table 7 reports demand estimates (in Panel A), and corresponding test results (in Panel B) for the three values of λ. While the nonlinear transformation of income makes a difference for some features of the demand system, test results are consistent with our main specification in Section 7, and strongly support Keystone as the better fitting model.

Table 7: Robustness to Box-Cox Transformations of Income in Demand

	RCNL Box Cox		
	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1.5$
Panel A: Demand Results:			
Price (in \$)	-0.352	-0.377	-0.478
	(0.034)	(0.019)	(0.088)
Package Size (= 3.5 oz)	0.121	0.134	0.119
	(0.037)	(0.000)	(0.047)
THC	0.027	0.029	0.029
	(0.006)	(0.007)	(0.008)
THC Squared	-0.001	-0.001	-0.002
	(0.000)	(0.000)	(0.003)
CBD	-0.044	-0.046	-0.048
	(0.009)	(0.009)	(0.010)
CBD Squared	-0.001	-0.001	-0.002
	(0.001)	(0.001)	(0.006)
Log Number Own Products	0.141	0.135	0.110
	(0.024)	(0.025)	(0.027)
σ	0.391	0.376	0.274
	(0.057)	(0.000)	(0.060)
Income × Constant	0.005	0.006	-0.004
	(0.001)	(0.000)	(0.004)
Income × Price	0.421	0.421	0.008
	(0.395)	(0.019)	(0.005)
Median Own Price Elasticity	-5.612	-5.352	-6.238
Median Aggregate Price Elasticity	-2.394	-2.356	-3.258
Diversion to outside option	0.391	0.401	0.468
Retailer FE	Yes	Yes	Yes
Processor FE	Yes	Yes	Yes
Year-Month FE	Yes	Yes	Yes
Panel B: Testing Results: Bertrand vs. Keystone, Tax Instruments			
T^{RV}	10.121	10.071	13.421
	***	***	***
F	38.5	32.8	43.0
	††† ^^^	††† ^^^	††† ^^^

Panel A reports demand estimates for a Box Cox transformation of income (measured in \$100,000). Columns 1-4 correspond to different calibrated values of the λ parameter in Equation (6). Panel B reports, for each value of λ, the RV test statistics T^{RV} and the effective F-statistic (Duarte et al., 2024) for testing Bertrand versus Keystone with ad valorem tax instruments. A positive RV test statistic suggests a better fit of Keystone. The symbol $***$ indicates rejection of the null of equal fit 0.01 confidence level. The symbols $\dagger\dagger\dagger$ and $\wedge\wedge\wedge$ indicated that F is above the appropriate critical values for worst-case size below 0.075, and best-case power above 0.95, respectively. Both T^{RV} and the F-statistics account for two-step estimation error and clustering at the market level. $n = 153,936$.

G.2 Robustness of Test Results to Alternative Cost Specifications

We consider the robustness of our testing results in Table 3 to alternative specifications of marginal cost. Results are reported in Table 8. Panel A reproduces the specification in the main text. Panels B and C include package size indicators, and consider alternative specifications of fixed effects in marginal cost. For all specifications, our preferred tax instruments are strong for size and power, and conclude for superior fit of the Keystone model. Different specifications of marginal cost affect the variation that is available for testing conduct. In particular, the specifications in Panels B and C incorporate both geographic and time fixed effects, thus absorbing considerable variation. This weakens instruments, especially RC and PC. Despite these additional hurdles, whenever the null is rejected, the test always rejects in favor of Keystone.

Table 8: Test Results for Different Levels of Fixed Effects

	Instruments:			
Statistic	Tax	RC	PC	WP
Panel A: Product Fixed Effects				
T^{RV}	13.39	4.52	9.88	16.79
	$***$	$***$	$***$	$***$
F	40.6	2.2	15.6	1,591.6
	†† † ∧∧∧	†† † ∧∧∧	†† † ∧∧∧	†† † ∧∧∧
Panel B: Retailer, Processor, and Year-Month Fixed Effects				
T^{RV}	2.39	-0.90	2.24	9.98
	$***$		$***$	$***$
F	4.2	0.5	1.4	4,912.6
	†† † ∧∧∧	†† †	†† † ∧∧∧	†† † ∧∧∧
Panel C: Retailer, and Year-Month Fixed Effects				
T^{RV}	2.74	-0.77	2.45	27.35
	$***$		$***$	$***$
F	4.1	0.5	1.4	3,232.1
	†† † ∧∧∧	†† †	†† † ∧∧	†† † ∧∧∧

The table reports, for each set of instruments, the RV test statistics T^{RV} and the effective F-statistic (Duarte et al., 2024) for testing Bertrand versus Keystone. Different panels correspond to different levels of fixed effects in marginal cost. A positive RV test statistic suggests a better fit of Keystone. The symbol $***$ indicates rejection of the null of equal fit 0.01 confidence level. The symbols ††† and ∧∧∧ indicated that F is below the appropriate critical values for worst-case size below 0.075, and best-case power above 0.95, respectively. Both T^{RV} and the F-statistics account for two-step estimation error and clustering at the market level. $n = 153,936$.

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